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SOME ASPECTS OF CIRCULAR RESTRICTED THREE-BODY PROBLEM FROM DIFFERENTIAL GEOMETRY POINT OF VIEW

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Abstract. This paper considers differential geometry methods for determining local geometrical parameters of zerovelocity curves (ZVC) and surfaces (ZVS) in the circular restricted three-body problem (CR3BP) and emphasizes some interesting characteristics. The obtained results indicate some principles in distribution of local geometrical parameters along ZVC and ZVS and their influence on orbital motion.

1. INTRODUCTION

Nowadays, papers that study three-body problem, mostly consider some specific cases like ZVS in CR3BP with variable masses (Luk'yanov, 1992), application to trojan asteroids (Bálint, 1978), capturing conditions (Szenkovits and Makó, 2005), and so on. Although CR3BP is the problem with a huge historical background, which occupied many great scientists including Newton, Lagrange, Gauss, Euler, Jacobi and many others, it is very difficult to find papers that consider fundamentals of this problem like the geometrical parameters of ZVC and ZVS and their influence on the orbit of the third body.

After some assumptions regarding units, ZVS are defined implicitly by the equation

$$f(x,y,z) = x^2 + y^2 + z^2 + 2\left(\frac{\mu_1}{\sqrt{(x+\mu_2)^2 + y^2 + z^2}} + \frac{\mu_2}{\sqrt{(x-\mu_1)^2 + y^2 + z^2}}\right) \quad (1)$$

Where C_J is the Jacobi constant (the only integral of motion of CR3BP) and μ_1 and μ_2 are gravitational constants of two large masses. The equation of ZVC is easily derived from the above equation by putting $z = 0$.

$$g(x,y) = x^2 + y^2 + 2\left(\frac{\mu_1}{\sqrt{(x+\mu_2)^2 + y^2}} + \frac{\mu_2}{\sqrt{(x-\mu_1)^2 + y^2}}\right) \tag{2}$$

From equations (1) and (2) can be seen that these curves and surfaces has complex analytical form and very complex geometry indeed, as it is shown in the figure below.

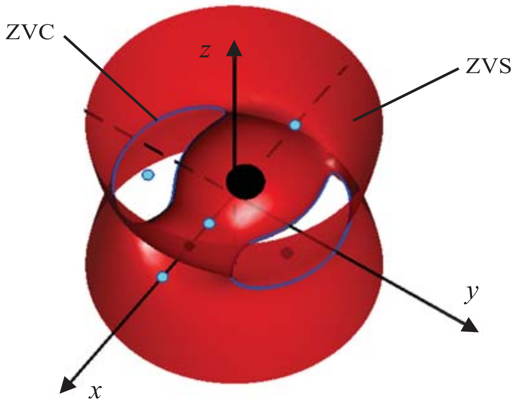

Figure 1. ZVS and ZVC.

2. METHODS

In order to find main geometrical characteristics of ZVC, ZVS and orbits, several numerical methods implemented in Matlab software were used. For determination of curvature of ZVC, a well known formula was used

$$\kappa = \frac{g_{xx}g_y^2 - 2g_{yy}g_xg_y + g_{yy}g_x^2}{\left(g_x^2 + g_y^2\right)^{\frac{3}{2}}} \tag{3}$$

where

$$g_x = \frac{\partial g(x,y)}{\partial x}, \quad g_y = \frac{\partial g(x,y)}{\partial y}, \quad g_{xx} = \frac{\partial^2 g(x,y)}{\partial x^2}, \quad g_{yy} = \frac{\partial^2 g(x,y)}{\partial y^2}, \quad g_{xy} = \frac{\partial^2 g(x,y)}{\partial x \partial y}.$$

For determination of main curvatures of ZVS, characteristic values of Hessian matrix were used

$$H = \begin{bmatrix} \frac{\partial N_x}{\partial x} & \frac{\partial N_x}{\partial y} & \frac{\partial N_x}{\partial z} \\ \frac{\partial N_y}{\partial x} & \frac{\partial N_y}{\partial y} & \frac{\partial N_y}{\partial z} \\ \frac{\partial N_z}{\partial x} & \frac{\partial N_z}{\partial y} & \frac{\partial N_z}{\partial z} \end{bmatrix}, \tag{4}$$

where N_x, N_y and N_z are components of normal unit vector defined by

$$\vec{N} = \frac{\vec{G}}{||G||}, \tag{5}$$

where

$$\vec{G} = \nabla f = \left[\begin{array}{ccc} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{array} \right]. \tag{6}$$

Two of three characteristic values of the Hessian matrix represent the main curvatures while the third one is equal to zero.

For determination of orbit of the third body, classical RK4 Runge-Kutta method was used to integrate equations of motion based on Newtons Low of gravitation. After discrete orbit of the particle was obtained, it was interpolated with the Lagrangian polynomial of the second degree in order to find curvature of the orbit. Since, the integration step in Runge-Kutta method was very small (1/1000 of the main bodies orbit) it was convenient to get good approximation of the real orbit by this method.

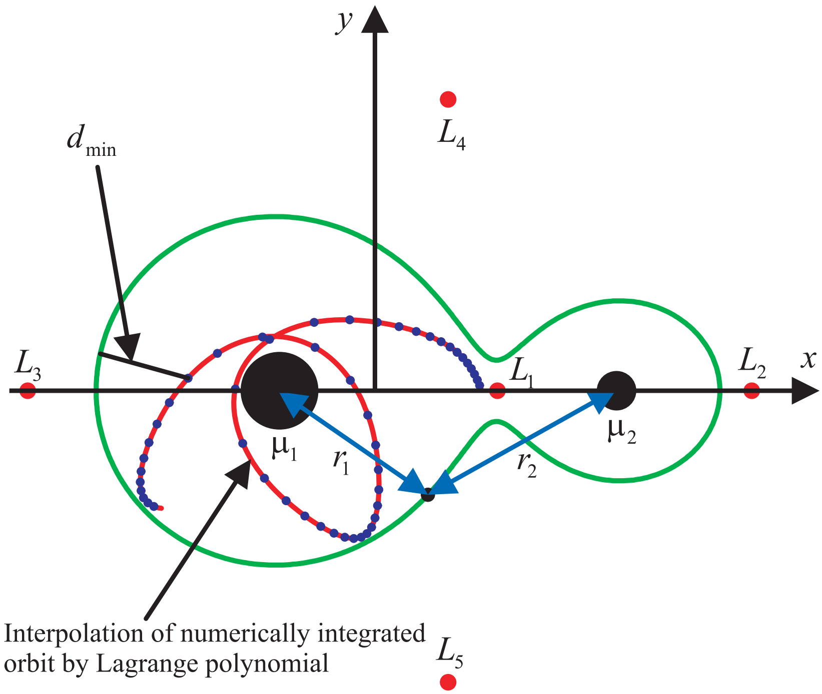

Figure 2. Orbit of the particle bounded by ZVC.

3. RESULTS AND CONCLUSIONS

In figures 3, 4 and 5 are shown typical distributions of local geometrical parameters of ZVS along ZVC. From these figures, it is obvious that the sum of distances from two large masses plays important role in geometry of ZVC and ZVS, because main, Gaussian and mean curvature have their local extremes in the narrow regions where this sum has its minimum.

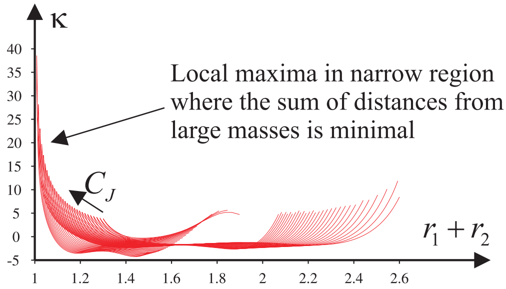

Figure 3. Typical distribution of main curvature along ZVC.

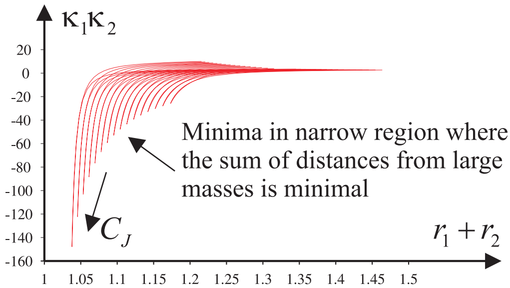

Figure 4. Typical distribution of Gaussian curvature along ZVC.

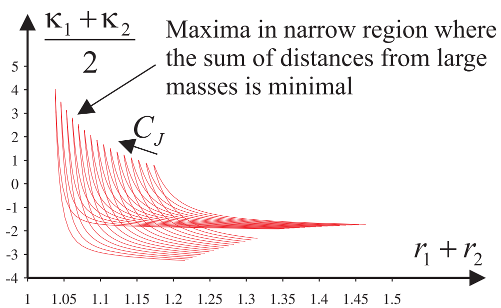

Figure 5. Typical distribution of mean curvature along ZVC.

In figure 5 is shown typical relation between curvature of the planar orbit and minimum distance from ZVC.

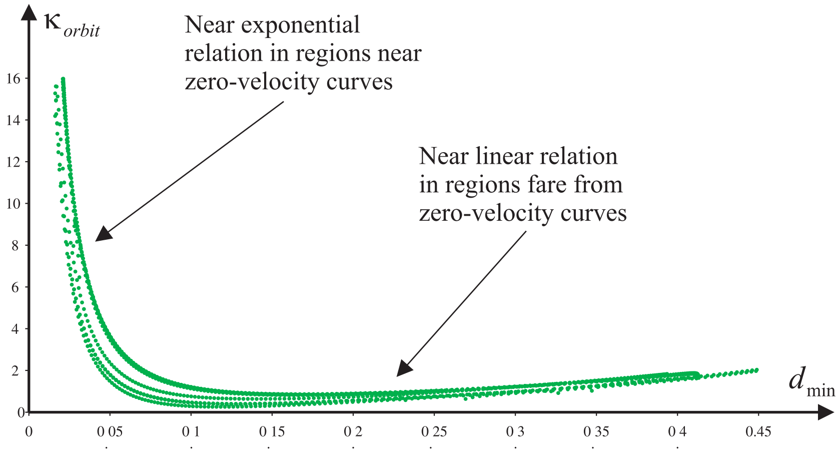

Figure 6. Typical relation between curvature of the planar orbit and minimum distance from ZVC.

From figure 5 one can se an interesting dependance of orbit curvature on minimal distance of the particle from ZVC. In regions near ZVC, orbit curvature has exponential decrease, while in the regions far from ZVC, orbit curvature has linear increase with minimal distance from ZVC.

From the all mentioned above, one can conclude that locations of the extreme values of the local geometrical parameters of ZVC and ZVC are related to the position of their points with reference to large masses. Also, local geometrical parameters of ZVC and ZVS may influence the local geometry of the orbit of the third body of the system.

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