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TOWARD A MODEL OF THE STELLAR INITIAL MASS FUNCTION FROM THE DENSITY DISTRIBUTION OF MOLECULAR CLOUD CLUMPS

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Abstract. Some basic steps toward creating a model of the Initial Mass Function (IMF) are proposed. The presented preliminary results include mass distributions of protostellar clumps, assuming a power relationship between their masses and densities ($\rho \propto m^{\gamma}$), and an approach for combined consideration of fragmentation and competitive accretion on the collapsing cores.

1. INTRODUCTION

Theoretical modeling of the stellar initial mass function (IMF) is a fundamental problem in astrophysics with a large range of implications: from cosmic reionisation and the formation of first galaxies, over the evolution and structure of our own Galaxy, down to the build-up of planets and planetary systems. The complicated physics behind remains, however, not deeply enough understood. Current observational and numerical work has begun to suggest that supersonic turbulent flows rather than static magnetic fields control star formation (Padoan and Nordlund 2002, Mac Low and Klessen 2004). The process of star formation from dense cores in molecular clouds (MCs) is often referred to as *gravoturbulent fragmentation*. Numerical simulations demonstrate that although supersonic turbulence can provide global support, it nevertheless produces density enhancements that allow local collapse (e.g. Heitsch, Mac Low and Klessen 2001, Li et al. 2004, Padoan et al. 2007). When a MC region of a few hundred solar masses or more coherently becomes gravitationally unstable, it contracts and builds up a dense clump highly structured in a hierarchical way, containing compact protostellar cores (Klessen and Burkert 2001; Clark et al. 2005).

Another theoretical approach to determine the IMF are analytical and semi-analytical models, based on assumptions that are drawn from the qualitative properties of numerical models or are direct results from numerical simulations. Padoan and Nordlund (2002) presented a semi-analytical model of the stellar IMF, considering specific properties of MC turbulence, shock jump conditions in the magnetized interstellar gas and their consequences for the formation of protostellar cores. Recently Dib, Kim and Shadmehri (2007) elaborated further the approach of Padoan and Nordlund (2002), including the rates of cores' coalescence and collapse.

Challenges to the theory of the stellar IMF reveal the necessity to revisit the existing models of gravoturbulent fragmentation. In this paper we present first preliminary results of our approach to develop a semi-analytical IMF model, based on several refined and more physical assumptions about the star formation process in the MCs.

2. BASIC ASSUMPTIONS OF OUR MODEL

2.1. Relationship "clump mass - clump density"

In the currently existing semi-analytical models of the IMF, no correlation between the masses of the protostellar clumps m and their mean densities n is assumed. The scaling of velocity in the model of Padoan and Nordlund (2002), assuming self-similar distribution of scales, leads to a power-law dependence between those quantities:

$$n \propto m^{\frac{\beta-1}{8-2\beta}} = m^{0.21} \text{ (MHD case)}$$
$$n \propto m^{(\beta-1)/(5-2\beta)} = m^{0.75} \text{ (MHD case)},$$

where β=1.9 is taken from numerical experiments. However, the largest numerical simulations of supersonic HD and MHD turbulence up-to-date (Padoan et al. 2007) show very large scatter of this relation (see their Fig. 5).

We have adopted a relation:

$$n \propto m^{\gamma} \tag{1}$$

for *all clumps*, prior to any requirement for gravitational instability. (The gravitationally collapsing clumps are usually labeled 'cores'.) Such relation could arise in a natural way from the relations of Larson (1981) for MCs with large span of sizes:

$$\langle n \rangle = 2000 \times \left(\frac{L}{1\ \text{pc}}\right)^{-1} \quad [\text{cm}^{-3}] \tag{2}$$

$$\sigma_v = 1.1 \times \left(\frac{L}{1\ \mathrm{pc}}\right)^{\alpha} \ \left[\frac{\mathrm{km}}{\mathrm{s}}\right], \tag{3}$$

where α=β-1)/2=0.4 (β≈1.8 from turbulence power spectrum, Padoan and Nordlund 1999). The clumps' density distribution is assumed to be lognormal, which is both a good approximation of the results of many numerical simulations (e.g. Padoan, Nordlund and Jones 1997, Passot and Vásquez-Semadeni 1998) and a basic assumption in the analytical model of IMF from gravoturbulent fragmentation of Padoan and Nordlund (2002):

$$p(\ln x)\, d\ln x = \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left(-\frac{1}{2}\left(\frac{\ln x + \frac{\sigma^2}{2}}{\sigma}\right)^2\right) d\ln x\,, \tag{4}$$

where $x=n/n_0$; n_0 and σ are, correspondingly, the mean density of the considered volume and the velocity dispersion at the given spatial scale L. A natural normalization of Eq. 1 could be done through the mean Jeans mass $m_{J,0}$:=m_0 for a clump that corresponds to $n_0 = \langle n \rangle$:

$$\frac{n}{n_0} = \left(\frac{m}{m_0}\right)^{\gamma}$$

$$m_{J,0} = 1.2\left(\frac{T}{10\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{1000\ \mathrm{cm}^{-3}}\right)^{-1/2}\ [M_\odot]$$

The physical conditions in the parent MC are implied in the clumps' density distribution (Eq. 4) mostly through the dependence of the velocity dispersion σ_v on magnetic field as the latter determines the Alfvén velocity v_A and the Alfvén Mach number $\mathcal{M}_A$. The average field B_0 could be estimated using output of numerical simulations (Padoan and Nordlund 1999):

$$B_0 = 8\mu G \times \left(\frac{n_0}{1000}\mathrm{cm}^{-3}\right)^{1/2}$$

Eventually only two parameters, the largest (starting) scale of the turbulent cascade L_0 and the exponent γ, determine the other mean features of the isothermal protostellar parent cloud. (The temperature is fixed at the typical value of T=10 K.) One can rewrite Eq. 4 for clump mass m as a variable and obtain the probability density function per logarithmic mass unit $x' = lg\left(\frac{m}{M_\odot}\right)$, i.e. the clump mass distribution:

$$p(x')dx' = \frac{\gamma}{lg(e)\sqrt{2\pi\sigma^2}} \times \exp\left(-\left(\frac{\left(x'/lg\,(e) + C(m_0)\right)\gamma + \sigma^2/2}{\sqrt{2\sigma^2}}\right)^2\right) dx' \tag{5}$$

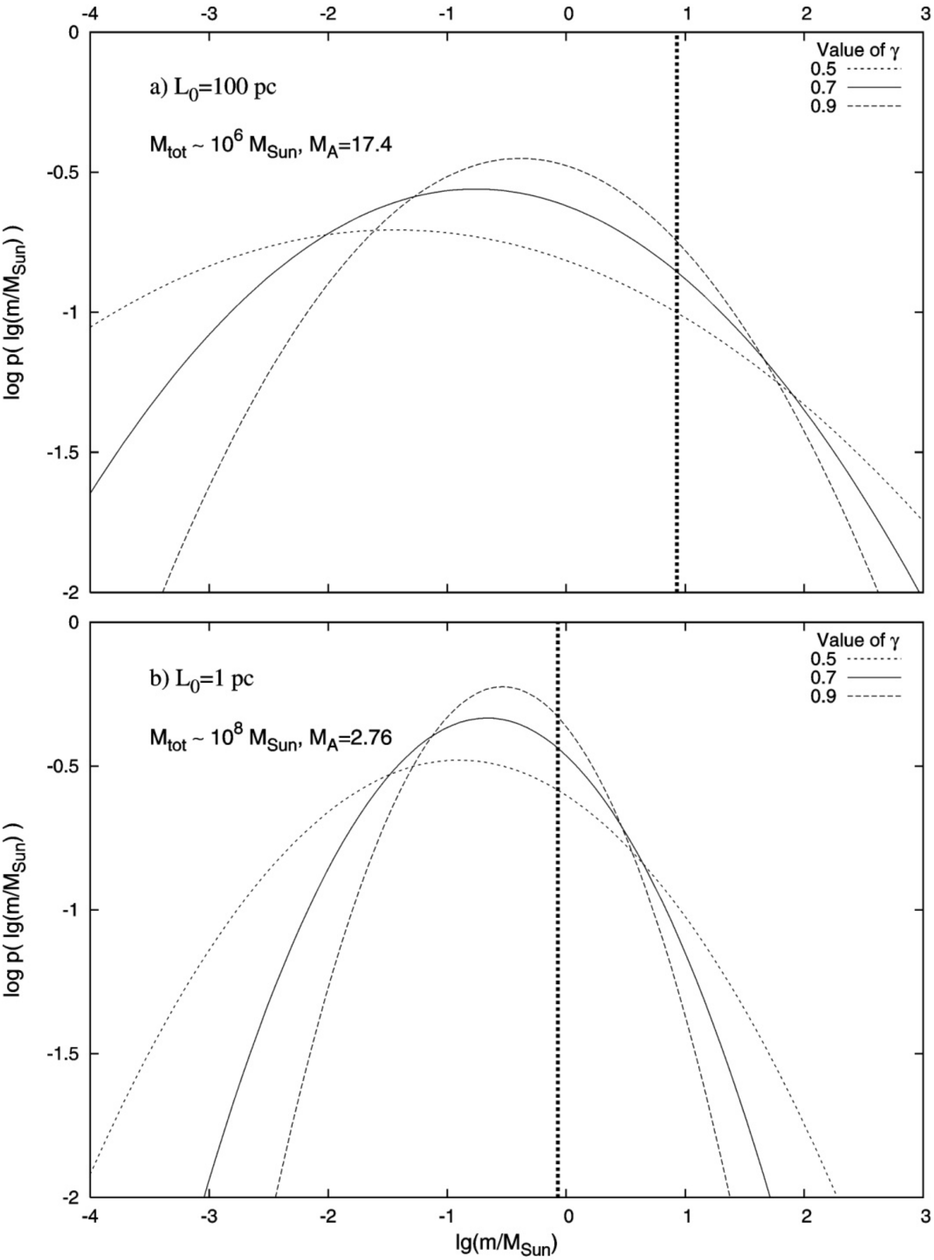

Figure 1: Clumps' mass distributions at two scales: a) *L*=100 pc (top), and b) *L*=1 pc (bottom). The total mass included and the corresponding Alfvén Mach number are given. The lower mass limit of clumps, satisfying the local Jeans criterion is shown (dash-dotted).

where m_0 corresponds to the mean mass density for a clump ρ_0 and $C(m_0) = \ln\left(\frac{M_\odot}{m_0}\right)$. The dispersion of the mass distribution σ is a function of $\mathcal{M}_A$ and σ_v and thus is scale-dependent. The turbulent cascade is supposed to run from L_0=100 pc down to the lowest possible (viscous) scale L ~ 0.1 pc, where $\mathcal{M}_A \sim 1$. The resulting clumps' mass distributions at two different scales and for 3 optional values of the exponent γ are shown in Fig. 1. As seen, a significant part of the clumps do not satisfy the local Jeans criterion for gravitational collapse. The location of the maxima of their mass distributions turn out to depend mainly on the chosen value of γ:

$$lg\left(\frac{m}{M_\odot}\right)_{max} = lg(e)\left(\frac{\sigma^2}{2\gamma} - \ln(m_0)\right) \qquad (6)$$

2.2. Relationship "core mass - masses of the formed protostars"

An usual but too strong assumption in the existing semi-analytical IMF models is that every MC core forms one star: 'protostellar cores' have been defined as objects of exactly one Jeans mass and their mass function is indeed a distribution of local Jeans masses m_J. Our approach is to consider the more general case, defining the efficiency of formation of such cores for each mass bin:

$$\epsilon = \epsilon(m, L)$$

Thus, for each scale L, scale-dependent IMFs are formed, centered around the value of the corresponding Jeans mass $m_J(L)$. Their superposition would give the desired IMF for the total mass, considered in the model. The characteristic core collapse timescale $\tau_{ff} \propto n^{-1/2}$ would depend on the Jeans mass for the corresponding turbulence scale $m_J(L)$ and will be included in the modeling of the scale-dependent IMFs for each mass bin.

2.3. Accretion/coalescence on/of protostellar cores

Dynamical interactions between protostellar cores may become important in rich compact clusters. The accretion rate of an individual core could be described as a function of the local density n and the square of the accretion radius (Bonnell et al. 2001). The latter quantity depends on whether the gas or the newly formed stars dominate the gravitational potential in a protocluster. Since these two regimes correspond to two stages of the protocluster evolution, the problem of timescale of accretion (evtl. coalescence) τ_{acc} (τ_{coal}) is introduced. In the model we are going to develop the accretion/coalescence on the protostellar cores will be treated together with the fragmentation (see above), taking into account the characteristic timescales τ_{ff} and τ_{acc} (τ_{coal}).

2.4. Modifications in the equation of state (EOS)

So far, any semi-analytical model of the relation between interstellar turbulence and the IMF has assumed an isothermal ideal-gas EOS. However, recent numerical work (e.g. Klessen, Spaans and Jappsen 2007) has demonstrated that even small changes in the EOS, i.e. in the thermodynamic response of the gas to external perturbations, leads to noticeable differences in the fragmentation behavior of the gas and consequently to a different IMF. Assuming an EOS in the form

$$P \propto \rho^{x},$$

we can take into account the dependence of the polytropic exponent x on the mean density n. Then, through numerical simulations, we can investigate how the varying EOS throughout the considered spatial scales may influence the resulting turbulent power spectrum.

3. FIRST RESULTS ON THE CLUMP MASS FUNCTION

We present current results on the intergrated over all scales clumps' mass function (CMF) that rest solely on our first basic assumption (Sec. 2.1). Each logarithmic mass bin has contributions from different turbulent scales and hence contains clumps, corresponding to different Jeans masses $m_J(L)$. Following Padoan and Nordlund (2002), a self-similar distribution is assumed at each scale L. That means that the number of clumps scales as $N \propto L^{-3}$ and, in view of Eq. 2, one obtains for the total mass $M_{tot}(L)$ at a given scale:

$$M_{tot}(L) = N(L)M(L) = \frac{L_0^3}{L^3}(nL^3) = \frac{L_0}{L}M_{tot}(L_0) \qquad (7)$$

Scaling the clump mass distribution with $M_{tot}(L)$, one derives the CMF at given L. Its cut-off limits (m_{down}, m_{up}) are obtained from the requirement:

$$M_{tot}(L) = \int_{x_{down}}^{x^{up}} 10^{x'} p(x') dx',$$

where x' and $p(x')$ are defined as in Eq. 5 and the integration is performed stepwise, symetrically from the mass distribution maximum (Eq. 6) until a value of $M_{tot}(L)$ is reached. The intergrated CMF is obtained as a superposition of CMFs for the whole range of scales. It is shown, for 3 chosen values of γ, in Fig. 2.

The integrated CMF is almost symmetrical and practically does not depend on the chosen value of γ, with a maximum is at $\sim 0.1 M_{\odot}$. The slope in the high-mass part is around the value for Salpeter stellar IMF (Salpeter 1955) that is currently believed to be universal phenomenon (e.g. Kroupa 2002, Chabrier 2003). We cau-

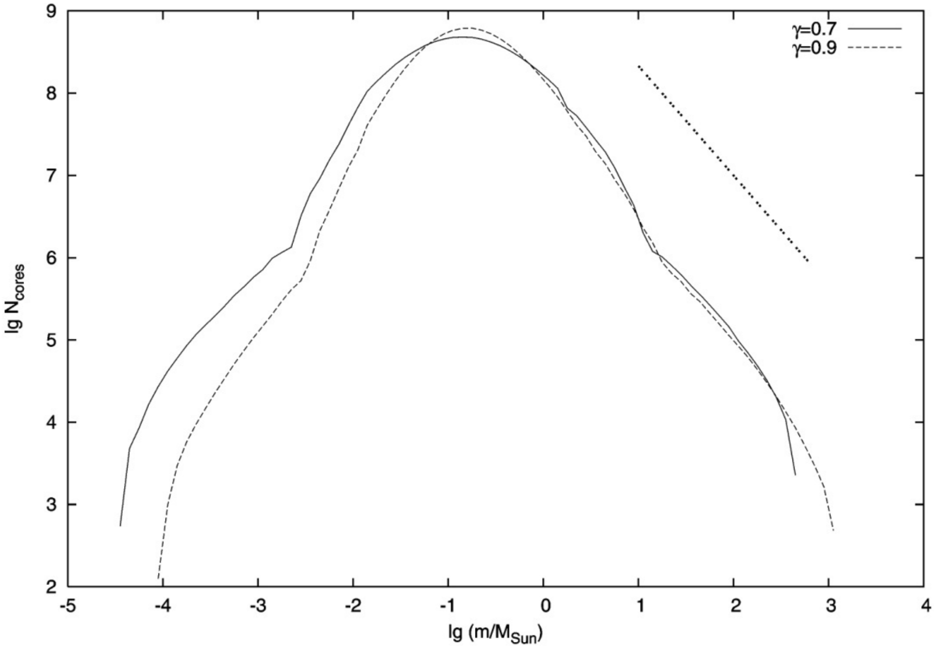

Figure 2: Integrated clumps' mass function for scales from L_0=100 pc to L=1 pc and 2 values of γ. The symbols are the same like in Fig. 1. The slope of a Salpeter IMF for massive stars is plotted for comparison (dotted).

tion, however, the reader that the application of our next 3 basic assumptions can change significantly this result. Most of the initially formed clumps are less massive than the local Jeans mass and both the accretion timescale and the star formation efficiency (Sect. 2.2 and 2.3) would play a crucial role for correct derivation of the IMF.

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