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ON THE FOUNDATION OF PHYSICS BASED ON TOPOLOGICAL SPACES: HILBERT'S SIXTH PROBLEM

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Abstract. Our paper is devoted to the solution of Hilbert's sixth problem, attempting to treat the problem of axiomatization of physics, as well as the foundation of physics as a formal mathematical theory. Within the framework of formal Set Theory, we have built a universe of topological spaces. On such grounds, a formal Space Theory is formulated. A definition of motion within a universe of topological spaces is introduced postulating that gravitational and other field effects could be predicted from it, and our final goal has been to found physics on new grounds.

1. INTRODUCTION

At the international congress of mathematicians held in Paris, 1900, David Hibert formulated his 23 open problems, as a challenge to and programme for mathematicians in the 20^{th} century. His sixth problem was concerned with the axiomatization of physics or the foundation of physics as a formal mathematical theory.

Hilbert's requirement for the formalization of physics is quite natural as, at the time, formalization of many mathematical theories started and continued throughout the 20^{th} century[1].

While today mathematics is represented by many formal theories which are based on the formal Set Theory, physics has always been based on different

[1] First of all, Hilbert founded geometry as a formal theory (Hilbert, 1930); furthermore, the appearance of logical paradoxes such as Russell's (Russell, 1906), and language paradoxes, such as Richard's (Richard, 1907), motivated mathematicians to formulate Set Theory in an axiomatic way and to define its language. In other words, Set Theory was founded as a formal mathematical theory (Zermelo, 1908; Jech, 1978). In such a way, Cantor's "naive" set theory had to be abandoned (Cantor, 1895).

grounds, thus failing to transform itself into a formal theory. Instead of the formalization process in physics, especially in the 20th century, we have a process of its geometrization.

The manner in which space was treated in physics led to its geometrization rather than its formalization. On the other hand, it is well known that topological spaces had not been defined until 1915. Riemman proposed his first topological ideas in *Inaugural Lectures* in the first half of the 19th century. Seventy years had to pass in order to arrive at a utilizable and viable definition of topological spaces. For the present state of topology, the major contributions have been made by Frechet, Hausdorff and Kuratowski. In 1906, Maurice Frechet introduced the notion of metric space by unifying the work on function spaces as proposed by Cantor, Volterra, Arzela, Hadamard, Ascoli, and others. A metric space is now considered a special case of general topological space. In 1914, Felix Hausdorff coined the term "topological space" providing the definition for what is now called the Hausdorff space (Hausdorff, 1914). In its current usage, a topological space is a slight generalization of Hausdorff spaces, given in 1922 by Kazimierz Kuratowski. Recalling these facts, it seems obvious that at the time when Albert Einstein was formulating his Special theory of relativity (1905) and afterwards the General theory of relativity (1915), it was not possible to work with topological spaces, simply because there was no precise definition of topology on sets (Einstein, 1905, 1952, 1916 and 1955). Einstein could only be familiar with the basic ideas of topological spaces, but without a precise definition it was impossible to go beyond. The manner in which physicists treated space was mainly based on geometrical spaces, i.e., sets on which geometrical objects, points, straight lines, and planes were defined, and on which a Euclidian or non-Euclidian geometry is realized, depending on the choice of the postulate of parallelism.

J. A. Wheeler, the founder of geometrodynamics (Misner and Wheeler, 1957, Wheeler, 1957; 1961), at a congress of philosophers in 1960, went one step further from geometrization and posed the following question (Grunbum, 1973): *"Is space-time only an arena within which field and particles move about as a "physical" and "foreign" entities or is the four dimensional continuum all there is. Is curved empty geometry a kind of building magic material out of which everything in physical world is made (1) slow curvature is one region of space a gravitational field (2) a rippled geometry with a different type of curvature somewhere else describes an electromagnetic field (3) knotted-up region of high curvature describes a charge and mass-energy that move like particles. Are field and particles foreign entities immersed in geometry or they are nothing but geometry"*. In 1972, at a conference on gravitation in quantization at the University of Boston, Wheeler found pure geometrization of physical phenomena insufficient and introduced the concept of pre-geometry. According to this concept, space-time is generated from a certain physical entity, from a space which carries geometry or space-generating geometry (Grunbum, 1973). In such a reformulated vision of physics, the notion of motion also had to undergo a proper "metamorphosis". Even though this was recognized as a problem, Wheeler failed to provide any solution.

A complete reduction of physics to space geometry or deduction of physics from space geometry could be one way of formalizing physics, due to the fact that geometry has the status of a formal mathematical theory. As a result, Hilbert's sixth problem would be resolved. Unfortunately, rather than deducing physics from space geometry, as the proper geometrization should do, a mere transcription was done of one part of the language of physics into geometrical language. Wheeler's concept of pre-geometry and the idea that geometry should be somehow generated highlighted the problem of the structure of carrying space and time, i.e., the structure that the determination of motion depends on. Finally, the *condition for axiomatization of physics should be determination of the structure of space and time, and, accordingly, a proper foundation of physics should be done on these re-established grounds.*

2. FOUNDATION OF SPACE THEORY

Every formal mathematical theory has four segments. The first and most important one is the language of formal theory. Starting from the language, we formulate a system of axioms. The axioms of a theory are assumptions about the existence of some objects in the theory or assumptions about the basic relations of the objects in the theory. The fundamental notions of a theory are always specified by a system of axioms. The third aspect of formal theories is some kind of the logic of the theory. These are mostly rules of derivation. In addition, we can speak about the fourth segment concerned with objects to which a theory is related.

Physics as a formal theory can be built up in two ways. It can be formulated as an independent formal theory, with a language, logic, system of axioms and objects which together should generate the whole "world" of physics. The second possibility is to found physics within the framework of an existing formal theory. In that case, the language of physics should be an extension of the language of this existing formal theory, the notions of physics should be derived from the notions of the existing theory, and the existence of objects to which physics is related should be derived from the existence of the objects in the starting theory.

In this paper, we choose the second route. We attempt to show that it is possible to build a formal Space Theory within the framework of ZFC Set Theory.[2] *Then we bring forth the arguments supporting the foundation of physics within our Space Theory.*[2]

But first, there are several questions that we need to clarify:

I. *What we assume by the foundation of physics?*

When we use the expression 'the foundation of physics', we mainly focus on the mathematatization of the notion of motion and a consistent inclusion of such a notion into mathematics. We define motion for every topological space in a given

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[2] Zermelo Fraenkel Set Theory with axiom of choice (Jech, 1978).

universe of topological spaces, with the underlying idea that so defined motion can help us predict gravitational and effects of the other fields.

II. For every *topological space, there is inherent motion.*

In general, a given set can be a space in many ways. Namely, every topology defined on a given set determines one space on a given set or makes a space out of that set. Hence, with the rising of topology, we are forced to abandon the idea about one possible space. In mathematics, when the notion of space is used, it generally refers to a topological space. Topological spaces are significant generalizations of geometrical spaces. Many important concepts on the nature of a unique underlying space, summarized in Adolf Grunbaum's *Philosophical Problems of Space and Time* (Grunbum, 1973)[3], become problematic as every assumption about the properties that such a space should fulfill *a priori* immediately leads to the limitations of the physical theory which we want to formulate.[4] In order to arrive at notions of physics in their full generality, one should start with space having the lowest number of *a priori* properties. These should be the main arguments for the idea that *physics might be based on general topological spaces.*

III. I*s there a need for Space Theory.*

If we want to build up physics on topological spaces, we are faced with another problem. Topology as a mathematical discipline is not built up as a formal theory.

It is well known that the axioms of Set Theory define the notion of set, one which all other terms of standard mathematics could be derived from or reduced to. Within Set Theory, the world of sets and set constructions has been built. The world of spaces and space constructions should be made within Space Theory. Objects to which Space Theory should refer must be topological spaces, i.e. sets with a defined topological structure. Space Theory should be one generalization of Set Theory, in the sense that topological spaces with a discrete topological structure could be considered sets.[5] *Therefore, the formal theory of discrete topological spaces should be the theory of sets.* This means that, in a discrete case, the language of Space Theory should be reduced to the language of Set Theory; in addition, some axioms of Space Theory should be reduced to the axioms of Set The-

[3] In one part of the book the author presents various conceptions of the nature of space and time, such as those proposed by Newton, Reimann Poincare, Eddington, Bridgman, Russell and Whitehead. It should be noted that all these scientists speak about one possible space and one possible time. Their conceptions are criticized in the second part of the book.

[4] That one should start from a space in building physics is very well in accordance with our intuition. It is hard to imagine physics without some underlying notion of space.

[5] A topological space X with a discrete topology can be identified with a set X. The closure of any subset A of a discrete topological space X is just A, so being in the closure can be reduced to membership relation. These simple facts are sufficient to show that a topologist sees sets as discrete topological spaces.

ory. *The idea of building a Space Theory is a fundamental one, as it implies interpolating the notion of space in the fundamentals of mathematics, so that the notion of set should be derived from the notion of space. In that way, we would be able to see sets as discrete topological spaces.*

Construction class spaces, in paper *Topologies on Classes* (Ćirić and Mijajlović, 1990), present a redefinition of the topological structure of sets, and allows one to define a topological structure of every element in a universe V of all sets.[6] In that way, the universe of sets becomes the universe of topological spaces, which we will denote as V_τ. The world of sets and set constructions thus becomes the world of spaces and space constructions.

The language of Set Theory, which describes sets, is now related to topological spaces, and the axioms of ZFC Set Theory, which state the existence of sets, now claim the existence of topological spaces. In other words, the ZFC Set Theory becomes the elementary Space Theory.[7] Predicates $\in$ and $=$ together with the symbols of logic, are sufficient to describe the behavior of sets and set constructions. As far as topological spaces and space constructions are concerned, it is natural to supplement the language of Set Theory with new predicates and axioms, in order to improve the notion of space and to obtain a richer world of spaces and space constructions. We therefore introduce predicates $\in_\tau$ and $=_\tau$ and are now able to define the language of Space Theory.

For every element $x \in V_\tau$, we can define its closure by using construction class spaces (Ćirić and Mijajlović, 1990). We denote it as $\overline{x}$ which is a set or a space of all elements of the universe of topological spaces which are near a space x. For spaces $x, y \in V_\tau$ we define:

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[6] All sets in universe of all sets we can arrange in the cumulative hierarchy of sets. By transfinite induction, we define:

$V_0 = \varnothing$

$V_\alpha = \cup_{\beta<\alpha} V_\beta$, if α is a limit ordinal.

$V_{\alpha+1} = P(V_\alpha)$, if α is not limit ordinal.

A set V_α have the following property (by induction):

(a) Each V_α is transitive.

(b) If $\alpha < \beta$, then $V_\alpha \subset V_\beta$.

(c) $\alpha \subset V_\alpha$.

The axiom of regularity implies that every set in some V_α, means that $V = \cup_{\alpha \in Ord} V_\alpha$.

[7] Fundamental relations between sets become relations between topological spaces. For example, elements of topological spaces are topological spaces as well, and elements of sets are sets, which, in general, is not the case in topology.

$$x \in_\tau y =_{def.} x \in \overline{y}.$$

Closure membership thus defines *topological membership relation* $\in_\tau$ which is reduced to the standard membership relation between sets in the case of discrete topologies on all sets of a universe of sets.

Topological membership relation $\in_\tau$ is defined via membership relation $\in$. Since membership relation $\in$ connects sets, the above definition has a sense only if the class $\overline{x}$ is a set, for every set $x \in V$. This can be easily verified. The relation $\in_\tau$ now connects topological spaces in the corresponding universe V_τ of topological spaces.[8]

In this way, we have a situation analogous to that in Set Theory, where the domain of membership relations is a universe V of all sets, so the *domain of* $\in_\tau$ *relations is the universe of topological spaces* V_τ.

Now that we have defined topological membership relation $\in_\tau$ and the universe of topological spaces V_τ, we are able to precisely formulate the idea about topological equality. For two topological spaces x and y which are the elements of a universe of topological spaces V_τ, x and y are said to be τ-equal if the following is fulfilled:

$$\forall z \in V_\tau \rightarrow x \in_\tau z \leftrightarrow y \in_\tau z,$$

which can be rewriten as:

$$x =_\tau y.$$

The definition of topological equality is analogous to the definition of set equality. One should note that in Set Theory two sets are by definition equal if all their collectivizing properties are equivalent, where a collectivizing property is one that collects elements into a set. Here we can continue and say that two topological spaces are τ-equal if all their collectivizing topological properties are equivalent.[9] It is well known that, due to Russell's paradox, we have the axiomatic foundation of Set Theory. If we want to build a formal Space Theory, then the language of

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[8]Here we define the relation $\in_\tau$ with the help of $\in$. A similar situation is found in set theory, where the relation = can be defined with the help of membership relation:

$$x = y =_{def.} (\forall z)(x \in z \leftrightarrow y \in z).$$

[9]We should note that the concept of τ-equality is not a natural one in topology. Namely, the basic idea is that τ- equality connects sets, especially those with a topological structure. Accordingly, such a relation cannot connect elements of a given topological space because the elements of topological spaces are usually sets, not topological spaces. Nevertheless, if we consider subsets of a given topological space as topological spaces with induced topologies, their τ- equality could be defined only if we have set topology on $P(X)$, which, generally, is not the case.

our theory must contain the relation $\in_\tau$ as a non-logical predicate and the formula $\phi[x]$: $x \notin_\tau x$ as one of the formulas of our theory. One question arises immediately: does the formula present a collectivizing or non-collectivizing property?
If we want to show that some property, expressed in terms of Space Theory is a collectivizing one, it is sufficient to show that such a topological property gathers elements into a set. For example, if we prove that property $\phi[x]$: $x \notin_\tau x$ does not collect elements into a set, then it does not collect elements into a topological space, either, and is, therfore, a non-collectivizing property.

The following statement holds: *Formula $x \notin_\tau x$ is not a collectivizing one, therefore it does not collect topological spaces x into a topological space.*[10]

The question of whether there are non-collectivizing properties expressed by topological membership relation is of the great importance. Taking into account that there are such properties, it is possible that some classes, specified by those properties, are spaces, so there is a possibility to have some axioms of Space Theory which are not amongst the axioms of Set Theory. Also, as the existence of some topological spaces depends on the choice of a starting universe of topological spaces, we can infer that *Space Theory has enough arguments to be founded and developed as an independent theory.*

Here we will present one way of how a formal Space Theory can be founded.[11]

The language of Space Theory L_τ can be defined as the language of ZFC Set Theory supplemented with predicates $\in_\tau$ and $=_\tau$ and with $x \in_\tau y$ and $x =_\tau y$ as atomic formulas.

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[10] *Proof.* Let (V, τ, σ) is topological class space, and V_τ corresponding universe of topological spaces. We denote with X class of all topological spaces x with following property $\phi(x)$: $x \notin_\tau x$. Let's prove that X is not a set and therefore it is neither topological space. Let's assume that it is a set. In that case, $\overline{X}$ is also a set, so relations: $\overline{X} \in X$ or $\overline{X} \notin X$ has sense and one of them holds certainly.

If $\overline{X} \in X$ holds, than set $\overline{X}$ will have property of all elements of set X, so it will be $\overline{X} \notin_\tau \overline{X}$, that means $\overline{X} \notin \overline{\overline{X}}$, while $\overline{\overline{X}} = \overline{X}$, it will be $\overline{X} \notin \overline{X}$, and that is in contradiction with assumption that $\overline{X} \in X \subset \overline{X}$, or $\overline{X} \in \overline{X}$.

If $\overline{X} \notin X$ holds, than $\overline{X} \in_\tau \overline{X}$, or $\overline{X} \in \overline{\overline{X}} = \overline{X}$, and also $\overline{X} \in \overline{X}$ which is impossible to holds for sets. So in both cases contradiction is proved, which means that assumption that X is topological space, and therefore and set is wrong.

[11] Practically, a formal theory of topological spaces.

Of special interest for investigations in Space Theory are the formulas which can be obtained by translating the axioms of Set Theory into the formulas of Space Theory, that is $\in$ into $\in_\tau$ and = into $=_\tau$. It may be that one of the formulas can be taken as an axiom of formal Space Theory, as all the formulas can be reduced to the axioms of Set Theory, provided all topologies on the universe of topological spaces are discrete. One of such formulas is a topological variant of the axiom of extensionality, which we will refer to as *topological axiom of extensionality*:

$$(\forall x)(\forall y)(\forall z)(z \in_\tau x \leftrightarrow z \in_\tau y) \to x =_\tau y,$$

which is equivalent to:

$$(\forall x)(\forall y)(\overline{x} = \overline{y}) \to \overline{\{x\}} = \overline{\{y\}},$$

The topological axiom of extensionality is also a formula of the language of Set Theory, which can hold or not for a given universe of topological spaces. In formulating Space Theory we can choose those universes in which the formula holds. If in a given universe of topological spaces the formula does not hold, we can take it as an axiom of Space Theory, on condition that the universe of topological spaces be reduced to those topological spaces where this formula holds, only if such a reduction is not a contradictory one.[12]

[12] There are examples of non-discrete universes of topological spaces in which the topological axiom of extensionality holds. Let α be a non-limit ordinal and set $X \subset V_\alpha - V_{\alpha-1}$, which means that it contains elements at one level. Let us assume that we have some non-discreet topological structure on X. Since $X \subset V_\alpha - V_{\alpha-1}$ holds, we have $X \cap P(X) = \varnothing$, where $P(X)$ is a partitive set of the set X. The relation $\overline{x} = \overline{y}$ is a relation of equivalence on P(X), which we can denote as θ. Let us take any T_1 topology on the factor set $P(X)/\theta$ and the weakest topology on $P(X)$, where the natural projection from $P(X)$ to $P(X)/\theta$ is continuous. We will have a topology on $P(X)$, so that for every two subsets A and B in the space B, the following implication holds:

$$\overline{A} = \overline{B} \text{ (in a space } X\text{)} \leftrightarrow \beta(A) = \beta(B) \text{ (in the space } P(X)\text{)}.$$

We will denote topological space X as X_α and $P(X)$ with $X_{\alpha+1}$ because X is bounded to the ordinal α. Continuing the same procedure for $P(X)$ as for X, we will come to $X_{\alpha+2}$, etc., if it is now γ first limit ordinal greater than α, we will put $X_\gamma = \cup X_\alpha$, for $\alpha \le \gamma$. Definitely, let us put $X = \cup X_\alpha$, where $\alpha \in ORD$, we will get topological class space (X, τ, σ) where τ and σ are classes of all open and closed sets in corresponding topological spaces X_α, for $\alpha \in ORD$. The topological class space (X, τ, σ) generate a universe of topological spaces V_τ in which topological axiom of extensionality holds.

For spaces $a, b \in V_\tau$ the transcription of the axiom of pairing is a statement that property $\phi(x): x =_\tau a \vee x =_\tau b$ is collectivizing, in that it gathers elements into a set and, therefore, into a topological space. Obviously, ϕ is a collectivizing property for every universe of topological spaces. It can be proved from the axioms of Set Theory. Similarly, topological formulation of the axioms of separation, union, partitive sets and substitution would be valid in any universe of topological spaces. That is not the case with the axioms of infinity and regularity. For example, transcription of the axiom of regularity, which here can be referred to as the topological axiom of regularity, can be formulated as:

Every nonempty space has a $\in_\tau$-minimal element.

$$\forall S[S \neq \varnothing \rightarrow (\exists x \in S)\overline{S} \cap \overline{x} = \varnothing]$$

Holding of the topological axiom of regularity implies holding of the axiom of regularity, so, instead of the axiom of regularity, we can take the topological axiom of regularity as an axiom of Set Theory. The universe of topological spaces can be chosen where the topological axiom of regularity holds or, as in Set Theory, the universe of topological spaces can be reduced to those spaces for which the topological axiom of regularity holds, if this reduction of the universe is not contradictory.

Definitely, as there are many set theories, there can be various space theories. While postulating and investigating these theories, one should come to a universe of topological spaces and a system of axioms which are the most viable for spaces and space constructions. Moreover, we should arrive at a Space Theory within which we will be able to build a world of spaces and space constructions in an optimal way. All this is beyond of scope of this paper.

The intention here was to accentuate a route which could lead to an acceptable Space Theory and, therefore, to a framework for an intuitive foundation of physics.

3. MOTION OF TOPOLOGICAL SPACES

Our notion of space, time and motion is based on our understanding of reality. Determination of these notions was and will be one of the greatest challenges in philosophy and fundamental sciences. The process of mathematization of categorial notions, which lasted for the whole 20th century, opens new possibilities to mathematics and, at the same time, forms mathematics as a kind of universal science which can be a powerful tool in understanding reality.[13]

[13] The great German philosopher Gottfried Wilhelm Leibniz, unambiguously demanded the foundation of a universal science and called for the formulation of a proper language. He proposed a project for its foundation and specified possible relations with mathematics. The realm of Aristotle's scholastic logic was found insufficient by Leibniz, and as an answer he proposed a new logic aiming to create a new method in epistemology. His efforts

While one of the categorial notions, namely, space, is mathematized through topology, the matematization of the notion of time or motion has not been done. The notions of motion and time lay far away from mathematics. Mathematical spaces are motionless, and motion is not their inherent property.[14]

Here, we will propose one possible mathematization of the notion of motion, one possible definition of motion of topological spaces.

1. *Motion of a given topological space should be derived from that space.*

In Set Theory we have a hierarchy of notions. All mathematical terms, at least in the so called standard mathematics, are derivable from the notion of set. The question of derivatation thus arises naturally, especially when we speak about the notion of motion. Is it possible for motion to be derived from the notion of space or can time be derived from the notion of space.[15] It was possible to pose such a question long before topology. Nevertheless, the possibilities of various spaces on a certain set, which topology opens, could now lead naturally to the plurality of times and motions. Furthermore, every space could generate its own time, its own motion and, in the final instance, its own physics.

2. *Motion of a given topological space should be a new topological space.*

In considering motion, we believe that there exists an underlying space which generates motion. We also refer to the self-evolution of motion, in some way a continuous transformation of space in itself by order in time, and we point to an entity which can be built up from space itself.[16] We see motion as a space proper-

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place him amongst the founders of mathematical and symbolic logic. The basic idea was to improve Aristotle's logic with the help of mathematics, and in a such a way to create an independent scientific discipline which would be a kind of universal mathematics or universal science. One of the favorite ideas of Leibniz was to create an alphabet of reasoning, which could serve as a deduction pattern for new ideas from the already existing ones, by applying proper deduction rules. For such a project, it was necessary to build a universal language, which would make communication among people much easier and, more importantly, it would make reasoning much simpler (Mijajlović, 1996).

[14]In classical physics we observe motion of objects or systems, and it is a consequence of a force exerted upon a system or an object. Motion is described in geometrical spaces as a continuous change in position. In quantum mechanics, we speak about the evolution of a system, where every possible state of a system is represented as a vector in the Hilbert's space and their superposition, and any change in the system is described via a physically relevant operator.

[15]We could say time as well, because if space generates time, it should generate many local times.

[16] It may sound strange that complete material for building up the motion of topological space can be found in a given space. We have such examples, when we define a certain topology on a given set, then complete material for building a space on a given set can be found in the set itself, because by choosing a certain collection of subsets of a given set, we define a space on the set. By starting with a set we end up with a space.

ty, derivable from the structure of space itself; we see it as existing and we derive its existence from the existence of space. In the world of Space Theory, there is nothing more than space.[17] If motion of a given topological space can be seen as a generalization of the model of space-time continuum, than we have even more reason to assume that motion of a given topological space is a new topological space. We consider topological spaces from a given universe of spaces, so the requirement that motion should be from the same universe is quite natural.[18]

3. *What is a natural relation between space and its motion*

Let X be a topological space which underlies motion. Motion of a topological space X, noted as $K(X)$, should be, according to the previous section, a new topological space. We bring forth the arguments that starting from the local homeomorphism $p : K(X) \to X$ it should be possible to derive some properties of motion $K(X)$ which are in agreement with our intuitive understanding of motion. Indeed, it can be assumed that every point $x \in X$ has a path $P_x \subset K(X)$. The elements of a path will be referred to as positions. In its motion, a point should pass "through" all its own positions, so it is natural to assume that a point and its positions have homeomorphous neighborhoods, so that $p^{-1}(x) \subset P_x$ holds.

Concerning the fact that we define motion in a certain universe of topological spaces and the topological axiom of extensionality holds within the framework of Space Theory, then points which are not topologically different should have the same paths, so the following should hold:

$$x =_\tau y \leftrightarrow P_x = P_y .$$

The idea that the path of a point can be one-dimensional, as far as topological spaces are concerned, should be abandoned and allow for paths of points to be multidimensional in the general case. Namely, in topological spaces we work with different dimensional functions which match each other only for some classes of topological spaces (Alexandrov and Pasinkoff, 1973).

It can be stated that there exists an "elementary part" of space or, so to say, a quantum of space $k(x) \subset X$ which is moving all the time together with the point $x \in k(x) \subset X$. For every element $y \in P_x$ there exist neighborhoods of points x

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[17]A similar situation applies to sets too, namely, in the world of set theory there is nothing more than sets themselves, and the existence quantor can refer only to sets. In the language of Space Theory, as in set theory, the existence quantor can refer only to spaces. Therefore, anything that is to be built something in Space Theory, e.g. motion, should presumably have the structure of a topological space.

[18]In a forthcoming paper, we have constructed the motion $K(X)$ of every space X from a given universe of space V_τ, so that $K(X) \in V_\tau$. In that way, we prove that motion can be derived from every topological space in a given universe.

and y, respectively U_y and V_y, so that $p \mid V_y : V_y \to U_y$ is homeomorphism. We can define:

$$k(x) = \cap_{y \in P_x} U(y).$$

All points which are topologically equal to $x \in X$ should be in the subspace $k(x)$. A quantum, in which the point $x \in X$ is given in general, is not uniquely determined.

4. *Motion of a given topological space should be topological invariant.*

Topologically speaking, we do not make a difference between two homeomorphous topological spaces. So, it is natural for homeomorphous topological spaces to have isomorphic motion. Namely, if X and Y are given topological spaces, $K(X)$ and $K(Y)$ are their motions and $p_X : K(X) \to X$ and $p_Y : K(Y) \to Y$ are corresponding local homeomorphisms, then for every homeomorphism $f : X \to Y$ there exists $f^* : K(X) \to K(Y)$ such that $f \circ p_X = p_Y \circ f^*$.

5. *What is local time.*

Local time is the order according to which a point "takes up" its positions on its own path. If we know the order which governs change in the position of a point, we can say that we know its movement. *Starting from time as a parameter, as is usually done in physics, and proceding to the order of positions on the path of a given point presents a natural generalization of, let us say, Euclidian notion of time, as far as topological spaces are concerned.*

Euclidian notion of time or time as a parameter is a special case of the above formulated concept of time. Indeed, let us assume that we have a straight line as a linearly ordered set via some order relation. If this order is complete, e.g., every bounded set has supremum and infinum, and if there is a countable subset of that straight line which is dense with respect to the order, then we say that this straight line is a real line, that it can be parameterized, and that its points can be considered as real numbers[19].

Local time should be in agreement with the topological relation of equality. Precisely, if $x \in X$ and P_x is the path of a point x, local time is any quasi order on the set P_x, denoted as $\preceq$ if the following holds:

$$((x \preceq y) \wedge (x =_\tau x', y =_\tau y')) \Rightarrow (x' \preceq y')$$

[19]That can be inferred from the ordering characteristic of real numbers, given through Cantor's theorems and the theorem about completion, (Jech, 1978), and Dedekind's geometrical definition of real numbers (Dedekind, 1932).

6. *Topology induced with* $K(X)$ *on* P_x *for every* $x \in X$ *is discrete.*

This is quite a natural condition, since a point $x \in X$ should continuously take up positions while it moves with respect to the topology of space $K(X)$.

Summarizing the conditions from 1-6, we arrive at the definition of motion in a given universe of topological spaces.

Finally, we are able to specify how physics can be founded as a formal theory. We start from the formal ZFC Set Theory, which is the basis for all mathematical theories. On a universe V of all sets, we choose a structure class space (V, τ, σ) and, in such a way, we come to a universe V_τ of topological spaces (Ćirić and Mijajlović, 1990). On the universe V_τ, we introduce the relations $\in_\tau$ and $=_\tau$ and we define the language of Space Theory. *According to the requirements of special physical theory which we want to build,* we have at our disposal in the language of Space Theory not only the axioms of ZFC system but also the possibility of formulating new axioms, such that some of the axioms of Set Theory can be replaced with new axioms. In such a way we obtain a formal Space Theory. For every topological space $X \in V_\tau$ we define its motion, or topological space $K(X) \in V_\tau$, local homeomorphism $p_X : K(X) \to X$, and local time on the path of every point in accordance with 1.-6.

We believe that construction of different motions of topological spaces in a given universe of spaces and their further investigation will contribute to the foundation of physics as a formal theory. Nevertheless, investigations of various space theories with the goal of finding a Space Theory and a universe of spaces which will generate motion and physical effects are no doubt the most important steps in the foundation of physics as a formal theory.

References

Alexandrov, P., Pasinkoff, B.: 1973, *Dimension Theory*, Nauka, Moskva.
Cantor, G.: 1895, Beitrage zur Begrilndung der transfiniten Mengenlehre, *Math. Ann.*, 46.
Ćirić, D. and Mijajlović, Ž.: 1990, Topologies on Classes, *Math. Balcanica*, New Ser. **4**, 10.
Dedekind, R.: 1932, *Gesammelte mathematische Werke*, 3. vol., Braunschweig (Vieweg).
Einstein, A.: 1905, On the Electrodynamics of Moving Bodies, *Annalen der Physik*, **17**, 891.
Einstein, A.: 1916, Die Grundlage der allgemeinen Relativitätstheorie, *Annalen der Physik*, 49.
Einstein, A.: 1952, *The foundations of the General Theory of Relativity*, The principle of Relativity, A Collection of Original Memoir New York Dover Publication Inc.
Einstein, A.: 1955, *The Meaning of Relativity*, Princeton Univ. Press.
Grunbum, A.: 1973, *Philosophical problems of space and time*, Riedl Publ. Co.
Hausdorff, F.: 1914, *Grundzuge der Mengenlehre*, Veil and Co., Leipzig.

Hilbert, D.: 1930, *Grundlagen der Geometrie* 1889, 7 edition, Leipzig-Berlin, 125.
Jech, T.: 1978, *Set Theory*, Academic Press New York, San Francisco, London.
Mijajlović, Ž.: 1996, Gedel's theorems, *Sci. Review*, **16**, 75.
Misner, Ch. and Wheeler, J.: 1957, Classical physics as geometry (Gravitation, electromagnetism, unquantized charge, and mass as a properties of curved empty space), *Annals of physics*, **2**, 525.
Reimann, B.: 1892, *Gesammelte mathematische Werke*, 2. ed., Leipzig (Teubner).
Richard, J.: 1907, Sur un paradoxe de la théorie des ensembles sur l'axiome de Zemelo, *L'Ens.Math.*, **9**, 94.
Russell, B.: 1906, Les paradoxes de la logique, *Revue de Méthaphysique et de Morale*, **14**, 627.
Šajković, R.: 1975, *Lajbnic i opšte dobro*, Prosveta, Beograd.
Wheeler, J. A.: 1957, On the nature of Quantum geometrodynamics, *Annals of physics*, **2**, 604.
Wheeler, J. A.: 1961, Geometrodynamics and the problem of motion, *Review of modern physics*, **33** (1), 63.
Zermelo, E.: 1908, Untersuchungen uber die Grundlagen der Mengenlehre, *Math. Ann.* **65**, 261.