Publl. Astron. Obs. Belgrade No. 73 (2002), 91 - 95 *Poster*

AN ANALYTICAL MODEL FOR DIFFERENTIAL SPECTRUM OF COSMIC RAYS

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Abstract. Galactic cosmic rays (GCRs) are the highest energy particle radiation to reach Earth. When GCRs enter our Solar System, they must overcome the outward – flowing solar wind. This wind impedes and slows the incoming GCRs, reducing their energy and preventing the lowest energy ones from reaching Earth. This effect is known as solar modulation. The Sun has an 11-year activity cycle which is reflected in the characteristics of the solar wind to modulate GCRs. As a result, the GCR intensity at Earth is anti – correlated with the level of solar activity, i. e., when solar activity is high - the GCR intensity at Earth is low.

Expression for differential cosmic ray spectrum $D(E)$ with smoothing function f containing tanh (*Tangens hyperbolicus*) is determined in this paper. The seven unknown coefficients in spectrum $D(E)$ are determined by means of Newton method. Results for different levels of solar activity are obtained.

Our analyses show that: 1. The ionization effect of the GCRs is inverse to the level of the solar activity (expressed by sunspot number, or Wolf number W) because of the solar wind modulation of the high energetic particles. 2. The GCRs have important influence on the solar-terrestrial relations during the solar minimum.

1. Introduction

Our atmosphere is continuously bombarded by a flux of high-energy primary Cosmic Rays (CRs). The cosmic rays radiation arriving at the orbit the Earth is composed of ≈98% nucleons stripped of all their orbital electrons, and ≈2 % electrons and positrons. The nuclear component consists ≈87% hydrogen, ≈12% helium and ≈1% of all the heavier nuclei from lithium to the actinides (Simpson, 1992).

The CRs spectrum spans a large range, from some MeV to 10^{21} eV. Toward low energies CR intensity is modulated by Solar Activity (Simpson, 1983). In the present paper will be modeled the differential spectrum (1.8 MeV – 100 GeV) of galactic cosmic ray with smoothing function tanh (*Tangens hyperbolicus*) (Velinov 2002, Buchvarova 2001, Velinov et al. 2001). In spectrum $D(E)$ are introduced eight coefficients. Using these coefficients we receive better correspondence between observed and experimental data (Hillas, 1972).

2. Cosmic ray spectrum

Here we use the following expression for the differential spectrum (energy range E from about 30 MeV to 100 GeV) of the galactic cosmic rays with account of the anomalous cosmic rays (energy range E from 1 MeV to about 30 MeV)(Velinov 2002, Buchvarova 2001):

$$\begin{aligned} D(E) = K\left(1+\tfrac{\alpha}{E}\right)^{-\beta}(1+E)^{-\gamma}\left(\tfrac{\tanh[s(E-\varepsilon)]}{2}+0.5\right)+ \\ +\lambda\left(1+\tfrac{\mu}{E^{\nu}}\right)\left(-\tfrac{\tanh[s(E-\varepsilon)]}{2}+0.5\right) \end{aligned} \tag{1}$$

This formula can be presented by exponential function also, using the relations between functions tanh and exponential function:

$$D(E) = K\left(1+\frac{\alpha}{E}\right)^{-\beta}\frac{(1+E)^{-\gamma}}{1+e^{-2X}}+\frac{\lambda}{1+e^{2X}}\left(1+\frac{\mu}{E^{\nu}}\right) \tag{2}$$

The new variable

$$X = (\tanh[s\,(E-\varepsilon)]) \tag{3}$$

is involved here. The coefficient s has value s = 40 (Velinov et al. 2001). The coefficients K, α, β, γ, λ, μ, ν and ε are solutions of the interpolation problem of this function through the points with the seven energy values 0.0018 GeV, 0.01 GeV, 0.023 GeV, 0.1 GeV, 0.39 GeV, 10 GeV and 100 GeV. These points are related also with the normalization conditions of spectrum (Velinov 1991). The value for γ was taken as constant, equal to 2.6 (Hillas 1972, Ginzburg and Syrovatskiy 1963). The calculation of the other parameters is treated by algorithm that combines the rapid local convergence of Newton's method with a globally convergent method for nonlinear systems of equations (Press et al. 1991). By inserting seven experimental measured points(E_i, D_i),one will get seven unknowns K, α, β, λ, μ, ν and ε

$$f_i[D_i(E_i), E_i, K, \alpha, \beta, \lambda, \mu, \nu, \varepsilon] = 0, i = 1, ...7.$$

Such nonlinear systems are treated by Newton - Raphson method in combination with line searches and backtracking. This algorithm is closely related to the quasi - Newton method of minimization. The strategy is quite simple: we always first try the full Newton step. However, we check at each iteration that the proposed step reduces f. If not, we *backtrack* along the Newton direction until we have an acceptable step. Because the Newton step is a descent direction for f, we are guaranteed to find an acceptable step by backtracking. The backtracking algorithm is discussed in more detail in (Press et al. 1991). The described program is realized in algorithmic language C.

3. Results

Table 1 contains a summary of the experimental data (E_i, D_i) for seven different levels of the energy E (Hillas 1972). For the energies $E < 390$ MeV a strong solar wind modulation of the cosmic rays intensity is observed. For each energy $E_i$in the interval between the energies 1.8 MeV ÷ 100 GeV eleven values of the solar activity level are given. Computation values of the coefficients K, α, β, λ, μ, ν and ε for eleven

Table 1: Data for protons/(m2.sec.str.MeV/nucleon) on seven energy (GeV) levels

	18E-4	10E-3	23E-2	10E-2	39E-2	10E+0	10E1
1	1.2645E+0	5.5217E-2	3.8924E-2	2.1553E-1	8.7000E-1	1.2300E-2	8E-5
2	1.4050E+0	6.0267E-2	4.1700E-2	2.2343E-1	9.0000E-1	1.2700E-2	8E-5
3	2.8100E+0	1.0100E-1	5.5600E-2	2.6294E-1	1.0000E+0	1.3900E-2	8E-5
4	4.2150E+0	1.5150E-1	8.3400E-2	3.4196E-1	1.1000E+0	1.5100E-2	8E-5
5	5.6200E+0	2.0200E-1	1.1120E-1	4.2098E-1	1.2000E+0	1.6300E-2	8E-5
6	7.0250E+0	2.5250E-1	1.3900E-1	5.0000E-1	1.3000E+0	1.7500E-2	8E-5
7	8.4300E+0	3.0300E-1	1.6680E-1	6.0000E-1	1.4000E+0	1.8700E-2	8E-5
8	9.8350E+0	3.5350E-1	1.9460E-1	7.0000E-1	1.5000E+0	1.9900E-2	8E-5
9	1.1240E+1	4.0400E-1	2.2240E-1	8.0000E-1	1.6000E+0	2.1100E-2	8E-5
10	1.2645E+1	4.5450E-1	2.5020E-1	9.0000E-1	1.7000E+0	2.2300E-2	8E-5
11	1.4040E+1	5.0500E-1	2.7800E-1	1.000E+0	1.8000E+0	2.3500E-2	8E-5

Table 2: Table 2. Coefficients K, α, β, λ, μ, ν and ε for curves 1 – 11 in the Figures 1 and 2

	K	α	β	λ	μ	ν	ε
1	22.553150	386.46588	0.347691	0.036069	0.000007	2.429358	0.118382
2	20.044742	245.44085	0.348596	0.038453	0.000008	2.4152250	0.118404
3	16.835422	109.29264	0.348861	0.047304	0.000029	2.2991270	0.117610
4	15.416599	62.021542	0.351491	0.071056	0.000028	2.2996740	0.115203
5	14.619727	38.608494	0.356963	0.094805	0.000028	2.2999020	0.113239
6	14.118236	24.972239	0.366218	0.118554	0.000028	2.3000380	0.111523
7	18.780476	16.264205	0.374302	0.142232	0.000028	2.2999110	0.109040
8	13.542305	10.374169	0.405133	0.165912	0.000028	2.2997980	0.106623
9	13.368533	6.2441400	0.446995	0.189606	0.000028	2.2997020	0.103899
10	13.238158	3.2969230	0.532534	0.213349	0.000028	2.2996190	0.099913
11	13.137741	1.2151520	0.799753	0.237266	0.000028	2.2992360	0.088612

values of the experimental data are shown in Table 2. They are obtained by means of the above mentioned Newton - Raphson method with line searches and backtracking (Press et al. 1991).

In the Figs. 1 and 2 are presented the results from the computation for the differential energy spectrum $D(E)$ of primary protons – the predominant component in the composition of the galactic cosmic rays and anomalous cosmic rays. Actually these curves are constituted on the basis of the results in Table 2.

The first four years of 11 - year solar cycle are characterized by increase of solar activity (SA) and the next seven years by its slow decrease (Velinov et al. 1974). In Fig. 1 are shown $D(E)$ during high and comparative medium SA levels, while Fig. 2 represents $D(E)$ during low and comparatively medium SA levels; curve 11 is for solar minimum. The other curves from the Figures give the spectrums $D(E)$ by different SA levels: curve 5 (Fig. 1) and curve 6 (Fig. 2) show a comparatively medium levels of the solar activity and curve 1 – the solar maximum.

4. Analysis of the results

The coefficients K and γ form the spectrum of the galactic cosmic rays. The

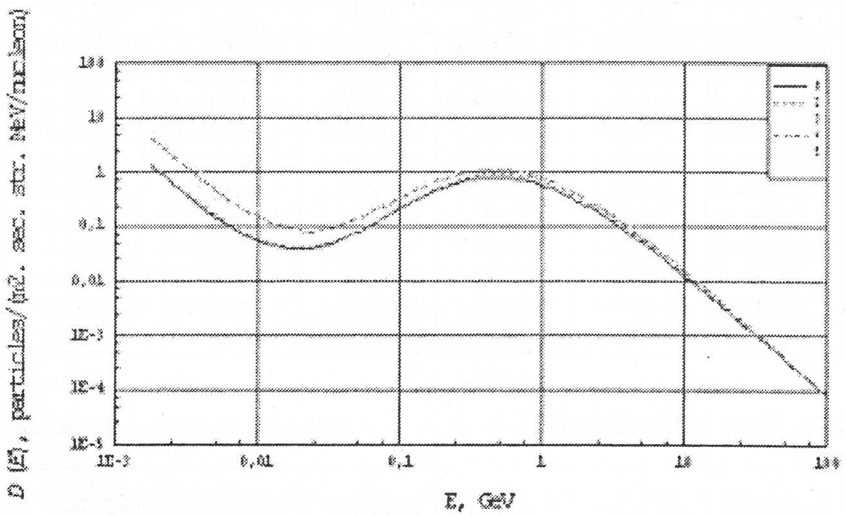

Fig. 1: The modeled differential spectrum D(E) of galactic CR protons and ACR for high and comparatively medium SA levels

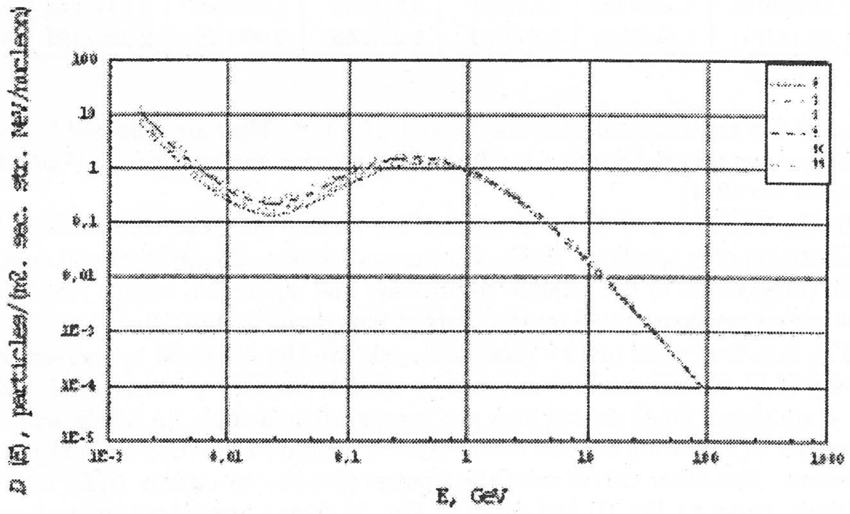

Fig. 2: The modeled differential spectrum D(E) of galactic CR protons and ACR for low and comparatively medium SA levels

parameters α and β show the influence of solar wind modulation on the primary cosmic ray spectrum. The coefficients λ, μ and ν relate to the anomalous cosmic rays.

The values of K, α and β remain constant in solar maximum (curve 1) and by average solar activity (curve 6). At the same time the coefficients λ and μ increase 3 – 4 times, while ν and ε are almost constant.

From the medium (curve 7) to minimum (curve 11) solar activity the values of $K$and α begin to decrease, while β remains almost constant. β increases twice only for solar minimum – curve 11. The coefficients λ, μ, ν and ε change very slowly in the transition from the medium to minimum solar activity.

$(1+E)^{-\gamma}$ In the interval $E \quad \geq 1$GeV the main contribution gives the term:

In the interval between the energies 30 MeV $\div$ 1GeV the first addend of the spectrum $D(E)$ gives the main contribution.

$\frac{\lambda}{1+e^{-2X}}\left(1+\frac{\mu}{E^{\nu}}\right)$ The energy range 1.8 MeV $\div$ 30 MeV is determined predominantly by the second addend

The term with tanh is smoothing function f, which is introduced in (Velinov 2002).

5. Conclusion

In this way, the modeled improved spectrum $D(E)$ presents well the 11-year variations of galactic cosmic rays which are one of the most important sources for ionization and dissociation in the Near Earth environment. We can determine the cosmic ray spectrum in the interval between the energies 1.8 MeV $\div$ 100 GeV for any level of the solar activity. The differential spectrum $D(E)$ can by used for computation of the electron production rate profiles and chemical balance in the ionospheric CR – layer (in the lower ionosphere 50 – 90 km) both for middle and high latitudes (Velinov et al. 1974). The present investigations are important also for the modeling of the ozone distribution in the middle atmosphere during different phases of solar activity (Tassev 1992ab).

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