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Abstract. A recently developed model of planetary formation (Balaž et al. 1999) allows a detailed investigation of various properties of planetary systems. In this paper we focus on the results regarding the spin of condensates, and show that the spin–mass relation obeys a power law with universal exponents. The results of the model are compared with the Solar system data, as well as with the binary star system data.

1. INTRODUCTION

There is a pressing need for the detailed modeling of the planetary accretion process. This has become even more apparent in recent years due to a wealth of new data regarding extrasolar planets (Marcy and Butler 1998). So far, the prevalent theoretical approach to planet formation has been in brute-force *ab-initio* simulations of large numbers of gravitating bodies (e.g. Ida and Makino 1992). Despite the enormous advances in computer speed, even the current generation of dedicated super-computers can not deal with the huge number of initial bodies ($N \geq 10^6$) needed to simulate the formation of a planetary system as a whole.

To achieve this goal, we have recently developed an effective model of planetary accretion utilizing a set of physical assumptions that simplify the accretion process considerably (Balaž et al. 1999). This has made possible the routine investigation of large numbers of initial particles up to $N = 10^{10}$. For each value of the parameters 100 independent runs were performed, in order to accumulate statistics.

Apart from N, the only parameter entering our effective model is the condensation parameter K, which is proportional to the ratio of masses of the protoplanetary disk and star. One of the main results of our work so far has been the uncovered independence of scaling exponents characterizing certain properties of the final condensates from the initial mass distribution in the protoplanetary disk. In this paper we will focus on the properties of exponents related to the spin of the condensates.

2. SPIN

Figure 1 depicts the typical result of our simulations for the relation between the spin of a condensate, i.e. angular momentum due to the rotation about its axis, and its mass. We have investigated a large number of initial mass distributions $\rho(r)$. In all cases we find a power law of the form $s \propto K^{\epsilon} m^{\omega}$. The scaling exponents ϵ and ω are almost independent of the choice of initial mass density. We have established the existence of two distinct classes of initial mass distributions. The above scaling exponents are universal inside each of those classes. Of the two exponents ω is obviously much more interesting, as it gives the connection between two directly measurable quantities.

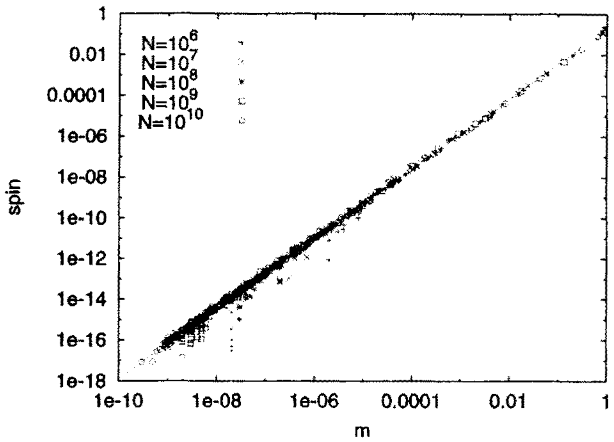

Fig. 1. Spins of condensates as a function of mass for $K = 0.1$ (best fit to Solar system) and $N = 10^6, 10^7, \ldots, 10^{10}$ initial particles. The results fit to $s \propto m^{\omega}$. Initial mass densities belonging to the first universality class all give $\omega = 1.75 \pm 0.03$, while those in the second universality class give $\omega = 1.94 \pm 0.06$.

In Fig. 2 we show several initial mass distributions belonging to the first universality class. All the distributions belonging to this class lead to the same values for the scaling exponents $\omega = 1.75 \pm 0.03$ and $\epsilon = 0.40 \pm 0.02$. Distributions belonging to the second universality class also, within error bars, lead to a single value for the scaling exponents. In particular, in this class we have $\omega = 1.94 \pm 0.06$. The simplest initial mass distribution in the second class is a generic uniform distribution from some minimal radius $r = a$ to a maximal radius $r = b$. The further analysis of these results, along with an analytic derivation of the two universality classes will be given in a future publication (Nađ–Perge et al.).

The corresponding data for the planets in the Solar system is given in Fig. 3. We see that our effective accretion model agrees quite well with phenomenology. The only two planets that do not satisfy the above spin-mass relation are Mercury and Venus. This is not surprising, as these are the two planets nearest to the Sun, where additional tidal lock effects, neglected in our simplified model, play an important role.

Let us note that our effective model represents a general model of gravitational condensation. As such it may be applied to other systems. For example, for large values of the condensation parameter K, the whole material in the protoplanetary disk condenses into a single object. This represents the scenario for the formation of a binary star system. In this limit the condensation process becomes independent of the order of condensation, making the model analytically solvable. The application of the model to binary star systems is still in progress. As a preliminary result, we used astrometric data on binary star systems (Malkov 1993) to plot the dependence of the orbital angular momentum L of a binary star system in terms of M_1 the mass of the primary. This is shown in Fig. 4.

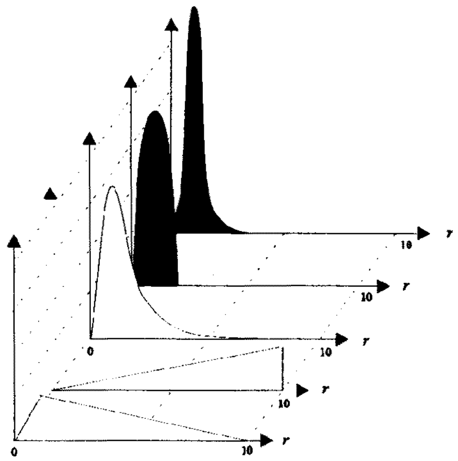

Fig. 2. Examples of initial mass distributions of the protoplanetary disk belonging to the first universality class.

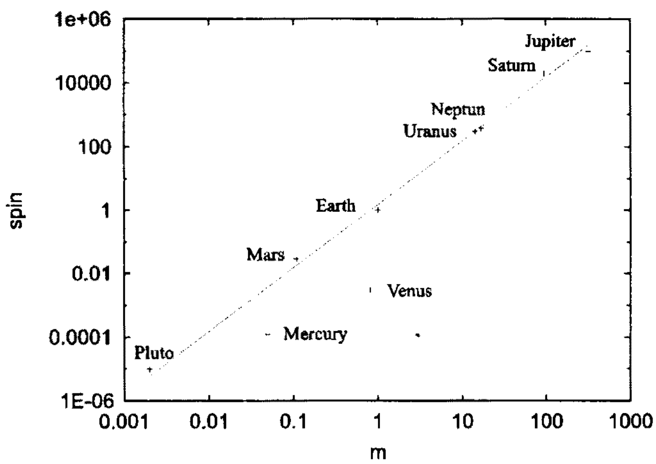

Fig. 3. Spin vs. mass for the planets of the Solar system. The planets satisfy $s \propto m^{\omega}$, where $\omega = 1.94 \pm 0.06$. As a result of tidal forces, Mercury and Venus do not lie on this curve.

From this figure we see that approximately 80% of binary star systems lie on the curve $L = 30 M_1{}^{1.75}$, while a further 10% (visual binaries) lie on $L = 850 M_1{}^{1.75}$. The majority of the remaining binary systems (mostly spectroscopic binaries) lie on a line of constant mass $M_1 \approx 1.16$ connecting the two curves. Under very simple assumptions it is possible to show that the spin of the secondary star is proportional to the orbital angular momentum. On the other hand, to a good approximation, binary systems satisfy $M_2 \propto M_1$, so that M_1 offers the only mass scale. Therefore, we see that binary star systems are well described by the scaling exponent $\omega = 1.75$, corresponding to the second universality class in our model. Although these results are preliminary it is interesting to see that both existing universality classes of our model are 'used' in nature: the one in the condensation of planetary systems, the other in the formation of binary stars.

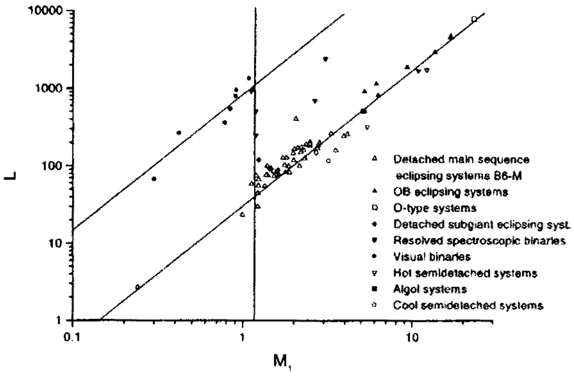

Fig. 4. Orbital angular momentum of a binary system about its center of mass as a function of the mass of the primary star.

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