Thermodynamics of fluid elements in the context of saturated isothermal turbulence in Molecular Clouds

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Main goal

- **We try to make use of powerful tools of the classical thermodynamics in order to investigate dynamical states of an hydrodynamical isothermal turbulent self-gravitating system.**

- **Our main assumption, inspired by the paper of Keto et al (2020), is that turbulent kinetic energy can be substituted for the macro-temperature of chaotic motion of fluid elements.**

- **As a proper sample for our system we use a model of turbulent self-gravitating isothermal molecular cloud which is at final stages of its life-cycle, when the dynamics is nearly in steady state.**

Molecular Clouds – the birth places of stars
Self-gravitating turbulent fluids

- How much is it important to understand the star-formation process?

- We can explain IMF, SFR, SFE and → the evolution of Galaxy

Molecular Clouds – Physical parameters and classification

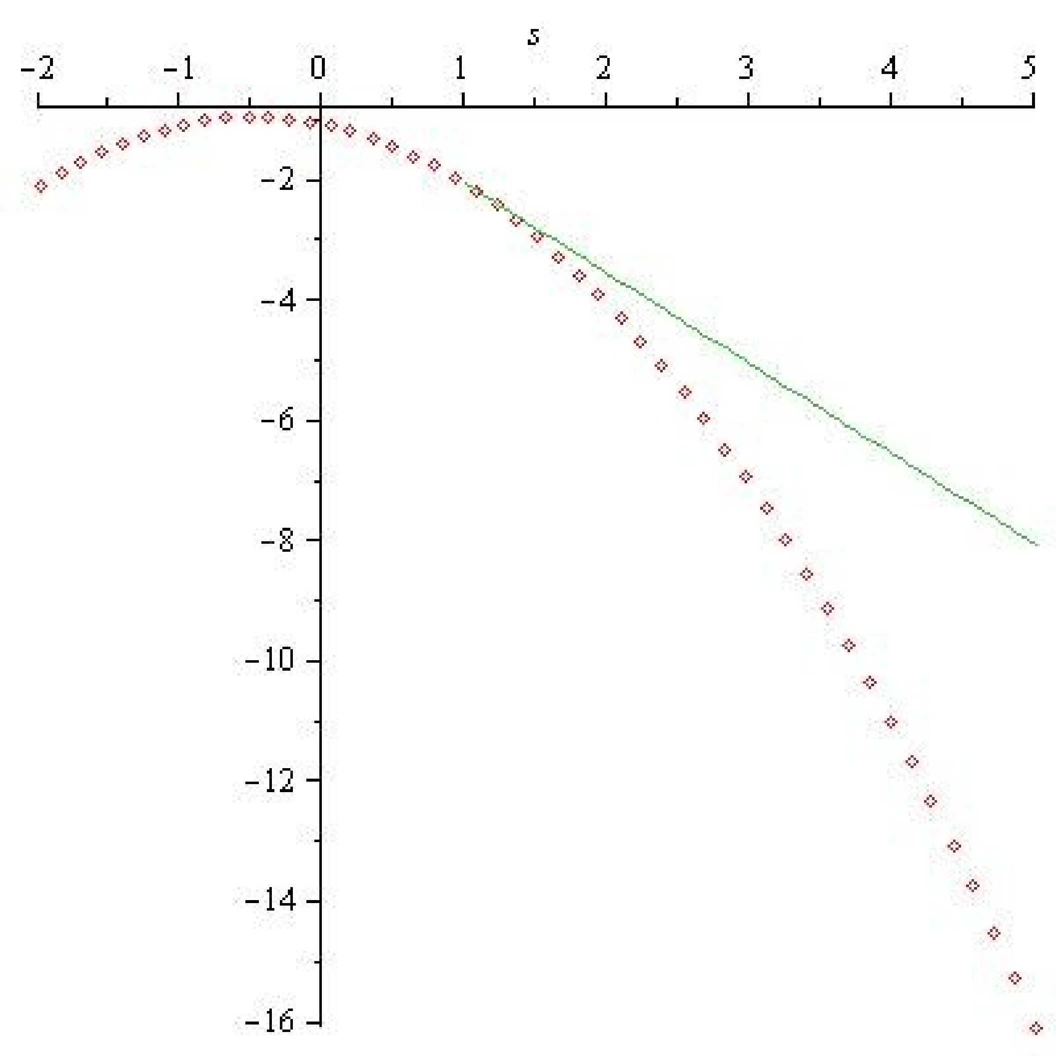
	GIANT MOLECULAR CLOUD COMPLEX	MOLECULAR CLOUD	STAR-FORMING CLUMP	PROTOSTELLAR CORE[a]
Size (pc)	10 – 60	2 – 20	0.1 – 2	$\lesssim 0.1$
Density ($n(H_2)/cm^3$)	100 – 500	$10^2 - 10^4$	$10^3 - 10^5$	$> 10^5$
Mass ($M_\odot$)	$10^4 - 10^6$	$10^2 - 10^4$	$10 - 10^3$	0.1 – 10
Line width ($km\ s^{-1}$)	5 – 15	1 – 10	0.3 – 3	0.1 – 0.7
Temperature (K)	7 – 15	10 – 30	10 – 30	7 – 15
Examples	W51, W3, M17, Orion-Monoceros, Taurus-Auriga-Perseus complex	L1641, L1630, W33, W3A, B227, L1495, L1529		see Section 4.3

[a] Protostellar cores in the "prestellar" phase, i.e. before the formation of the protostar in its interior.

PDF of mass-density

Lognormal – turbulence (isothermal)

PL-tail- turbulence and gravity



$$\text{lognormal} \propto \exp\left(- \frac{\left(s - \bar{s}\right)^2}{2\sigma^2} \right)$$

$$s = \ln\left(\rho / \rho_0\right)$$

$$\text{PL-tail} \propto \exp\left(qs\right); \quad q < 0$$

PDF of mass-density

Lognormal – turbulence (isothermal)

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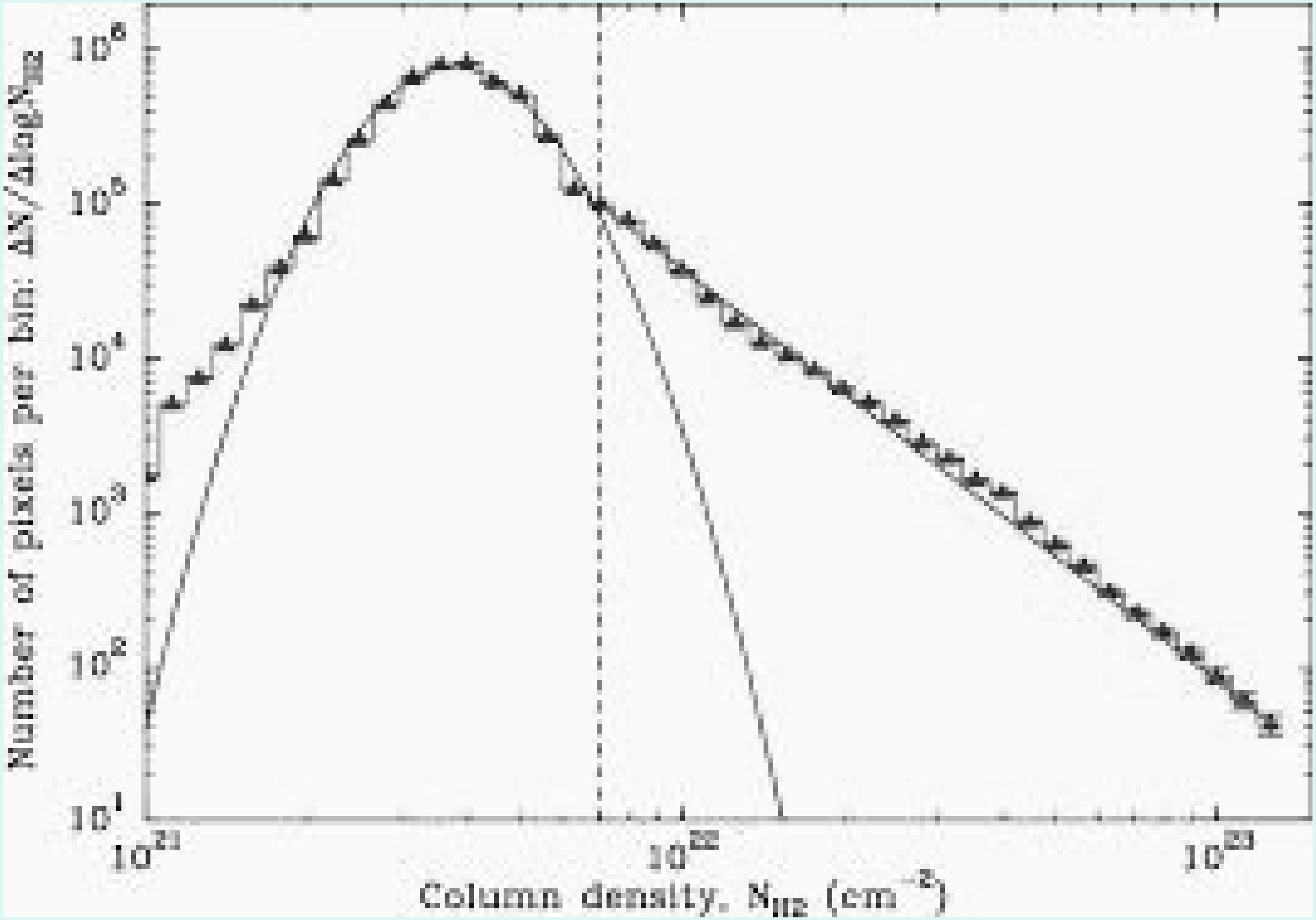

The abstract scale

$$l(s) \equiv l_c \left[\int_s^\infty p(s')\, ds' \right]^{1/3}$$

Our model – the MC physics

- Turbulence – fully developed and saturated; there exists an inertial range of scales: $l_d \leq l \leq l_{up}$
- Gravity – self-gravity and gravity from the surrounding medium
- Thermodynamics – isothermal equilibrium
- Magnetic fields and feedback from young stars are neglected

The turbulence locally is homogeneous and isotropic → the motion of fluid elements is purely chaotic

→ ***This local motion can be modeled as a perfect gas of fluid elements***

Our main idea

- Turbulent kinetic energy ←→ Macro temperature of the chaotic motion of fluid elements (Keto et al. 2020)

$$\frac{1}{2}m\sigma\left(l\right)^{2} \equiv \frac{3}{2}\kappa\theta\left(l\right)$$

m - *the mass of a fluid element*

$\sigma(l)$- *the turbulent velocity dispersion*

Our model

- We regard at every scale
  a physically small (homogeneous) volume

$$l_d \leq l \leq l_c \quad , \quad V_0 << l^3$$

- The gravitational potential in this volume

$$\varphi(l) = \varphi_s(l) + \varphi_m$$

$$\varphi_s(l) \text{ - } \textit{the self - gravity}$$

$$\varphi_m \text{ - } \textit{the potential due to surrounding medium}$$

Model – the internal energy of the small volume

$$\varepsilon\left(l\right)=\frac{3}{2}N\left(l\right)\kappa\theta\left(l\right)+\frac{3}{2}\frac{m}{m_0}N\left(l\right)\kappa T+mN\left(l\right)\varphi\left(l\right)$$

Equations – the entropy 1

$$d\varepsilon = \frac{3}{2}N\kappa d\theta + \left(\frac{3}{2}\kappa\theta + \frac{3}{2}\frac{m}{m_0}\kappa T + m\varphi\right)dN$$

$$d\varepsilon = \theta ds - PdV_0 + \mu dN \quad - \; First\;Law$$

$$V_0\;and\;N\;are\;const.$$

$$\Rightarrow \quad ds = \frac{3}{2}\frac{N\kappa}{\theta}d\theta$$

Equations – the entropy 2

$$\Rightarrow \quad s\left(\theta, N\right) = \frac{3}{2} N\kappa \ln\left(\theta / \theta_d\right)$$

θ_d - *the macro - temperature at disipation scale* l_d

Third Law:

$s(\theta_d, N) = 0$ - *the entropy at disipation scale* l_d

Equations – the free energy

$$f(\theta,N)=\varepsilon(\theta,N)-\theta s(\theta,N)$$

$$\Rightarrow \quad f(\theta,N)=\frac{3}{2}N\kappa\theta\left(1-\ln\left(\theta/\theta_d\right)\right)+$$

$$+\frac{3}{2}\frac{m}{m_0}N\kappa T+mN\varphi$$

Equations – the Gibbs energy

$$g(\theta, N) = \varepsilon(\theta, N) - \theta s(\theta, N) + PV_0$$

$$\Rightarrow \quad g(\theta, N) = \frac{3}{2} N\kappa\theta \left(5/3 - \ln\left(\theta / \theta_d\right)\right) +$$

$$+ \frac{3}{2}\frac{m}{m_0} N\kappa T + mN\varphi$$

$$PV_0 = N\kappa\theta \quad - \quad Clapeyron - Mendeleev\ equ.$$

$$for\ the\ macro - gas$$

Stability analysis of the system

- Boundary conditions for the cloud –
  fixed macro-temperature, pressure, and number of fluid elements

- The small physical volume is set at the same conditions in regard to its surrounding medium → hence it is a ***grand canonical ensemble***

- Therefore the relevant potential is the Gibbs energy

$$g = g\left(\theta, N, V_0\right)$$

Stability analysis of the system

- Starting from the Gibbs energy set in a non-equilibrium form

$$g\left(\theta, N, V_0\right) = \varepsilon\left(\theta, N\right) - \theta_0 s\left(\theta, N\right) + P_0 V_0$$

- We take the partial derivatives in regard to macro-temperature and volume and set them to zero to obtain the conditions for extremum

$$\left(\frac{\partial g}{\partial \theta}\right)_{N,V_0} = 0 \rightarrow \quad \theta = \theta_0 \quad ; \quad \left(\frac{\partial g}{\partial V_0}\right)_{\theta,N} = 0 \rightarrow \quad P = P_0$$

Stability analysis of the system

- The kind of the extremum depends on the sign of the functuonal determinant D (which elements are the second partial derivatives calculated at the extremum point)

$$D = \frac{3}{2}\left(\frac{N\kappa}{V_0}\right)^2 > 0$$

- D is positive → hence the Gibbs energy has a minimum → ***The system (small volume) resides at a stable dynamical state***

Stability analysis of the system

- What can one conclude for the whole cloud?

-> We have obtained the cloud's medium is locally dynamically stable, and hence the macro-temperature and pressure change continuously through the fluid, then large parcels of the cloud will be stable.

-> For the whole cloud the latter conclusion will be valid if the macro-temperature and pressure change through the cloud boundary without jumps.

Conclusions

This novel approach, inspired by the work of Keto at al. (2020), shows the ability of the classical thermodynamics to give a fiducial description of the equilibrium dynamical states of one hydrodynamical isothermal turbulent self-gravitating system, represented here by a molecular cloud model.

Despite of several approximations concerning the presented physical picture we consider our attempt as a sensible step in this direction.