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THE OPTICAL PROPERTIES OF HYDROGEN PLASMA IN THE FRAME OF THE FULLY QUANTUM METHOD BASED ON A CUT-OFF COULOMB MODEL POTENTIAL IN DIPOLE APPROXIMATION

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Abstract. The absorption coefficients of hydrogen plasma, calculated within the frame of cut-off Coulomb potential model, for the wide area of electron densities and temperatures observed within the solar atmosphere are presented here. The optical parameter of hydrogen plasma of mid and moderately high nonideality parameter could be described successfully, thus enabling the modeling of optical properties, especially the calculation of plasma opacity. The model was proven in both convergence towards normal condition, ideal plasma case, as well as with the help of analysis of the experimental data and further theoretical consideration. The model potential is solvable in entire space and within entire energy spectrum, thus the yielded wave function solutions are a combination of a special functions. The special form of the cut-off Coulomb potential, possesses a unique feature that enables the precise, fully quantum method of calculation of inverse Brehmstrahlung effect. The method covers the plasma with nonideality coefficient Γ from 0.0005656 for $N_e = 2{\times}10^{18}$ cm^{-3} and T = 6 MK up to 2.69 for $N_e = 2{\times}10^{21}$ cm^{-3} and T = 10 kK, from non interacting plasma up to the systems with the strong Coulomb coupling. Although the presented method development is still a work in progress the possibility of unifying a mode for both transport and optical properties of plasma within same model is an attractive direction for its further development.

1. INTRODUCTION

In the theoretical models of Solar plasma a problems of plasma opacity, energy transport as well as radiative transfer under moderate and strong non-ideality are of strong interest. For the deeper analysis of the subject reffer to Fortov & Iakubov

(1999), Rogers & Iglesias (1998), Mihajlov et al. (2011, 2013). The strong coupling and density effects in plasma radiation were the subject of numerous experimental and theoretical studies in the last decades. The presented quantum way of describing atomic photo-absorption processes in dense strongly ionized hydrogen plasma is based on the approximation of the cut-off Coulomb potential. By now this approximation has been used in order to describe transport properties of dense plasma (see e.g. Fortov & Iakubov (1999), Mihajlov et al. (1989), Ignjatović et al (2017)), but it was clear that it could be applied to some absorption processes in non-ideal plasmas too Mihajlov et al. (2011), Mihajlov et al. (2011, 2015), Sakan et al. (2005). More detailed explanation could be find in Sakan et al. (2018).

2. THEORY

2.1. The approximation of the cut-off Coulomb potential

Many body processes could be described by the use of transformation to the corresponding single-particle processes in an adequately chosen model potential, For the detailed theory Sakan et al. (2018) should be considered. As an adequate model potential for hydrogen plasma of higher density, for instance reffer to Mihajlov et al. (1989), Mihajlov et al. (2011), the screening cut-off Coulomb potential, that satisfies above conditions, and is used here is in form

$$U_c(r) = \begin{Bmatrix} \frac{-e^2}{r} + \frac{1}{r_c} & : r \leq r_c \\ 0 & r > r_c \end{Bmatrix} \tag{1}$$

where the mean potential energy of an electron in the considered hydrogen plasma $U_c = -e^2/r_c$ is used as an energy origin of the potential. Here e is the modulus of the electron charge, r - distance from the ion, and cut-off radius r_c - the characteristic screening length of the considered plasma.

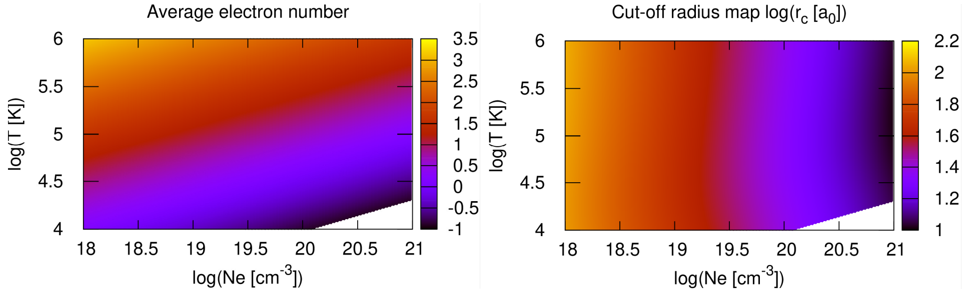

Figure 1: Left figure, the expected area of good agreement of the theoretical model, average electron number within screening sphere. Right, the expected area of good agreement of the theoretical model, logarithm of average electron number within screening sphere.

The cut-off radius r_c can be determined, see details in Mihajlov et al. (2009). The code that calculates the characteristic screening length of considered plasma uses electron density N_e, and temperature T, and is given at *https://github.com/nsakan972/ESPM-16.git*, it is free for use.

The model is expected to have good agreement with the modeled plasma conditions in the area where there is no many electrons within Wigner-Seitz sphere of screening radius r_c, and where the cut-off radius is smaller of few hundreds. The maps of the agreement could be seen on Figure **1**. In accordance with the presented data it is expected to have a good agreement of the model and theory within the range of electron densities $5{\times}10^{18}$ $\mathrm{cm}^{-3} \geqq N_e \geqq 10^{20}$ cm^{-3}, and temperatures in range 10^4 K $\geqq T_e \geqq 10^6$ K. Such ranges could be related to Solar photosphere and part of the chromosphere, please refer to Fig. **1**. on page 347. in Massey (1982). Although there are evidences that the model describes the plasma of lower electron density numbers and lower temperatures, in such cases a more detailed models exist, and in our case a comparison that was carried out serves as a simple proof of validity of model.

The main quantyty that describes the non-ideality of plasma is a coefficient that presents a ratio of Coulomb energy to the termal energy, in accordance with Fortov & Iakubov (1999) is given by $\Gamma_{ee} = e^2\beta/r_{WS}$, where $\beta = 1/(k_B\ T)$, and r_{WS} is Wigner-Seitz radius. The values for Γ_{ee} are merrit for the Coulomb interaction importance, e.g. is it a close to ideal system or the plasma goes to strongly couplet coulomb system.

3. THE CALCULATED QUANTITIES

In accordance with previously mentioned theory the behavior of the dipole matrix element for the bound-bound transitions is investigated, and the results are presented here. The dipole moment is given by $D(r;r_c;n_i,l_i;n_f,l_f) = \langle n_f,l_f|\ \mathbf{r}\ |n_i,l_i\rangle$, where $|n_i,l_i\rangle$ and $|n_f,l_f\rangle$ are initial and final state wave functions obtained within the model of cut-off Coulomb potential. So all the photon emission and absorption parameters are related with the dipole matrix element. A essential calculation for this is the solving of the radial part of Schrodinger equation, e.g. finding of energy level values as well as the radial parts of the wave functions. The radial part, $R_{n,l;rc}(r)$ presented as $\chi(r) = r\ R(r)$ for the selected level n=3 and l = 0 as well as n=3 and l = 1 could be seen on Figure 2. left and right side figure respectively. Please note that in the strong local plasma field a wave functions differ significantly from Coulomb case and as such reflect on all parameters and further calculated values. The dipole moments for hydrogen atom without plasma influence are equal to the values from Hoang-Binh (2005). The behavior of dipole moments is investigated under the modeled plasma influence.

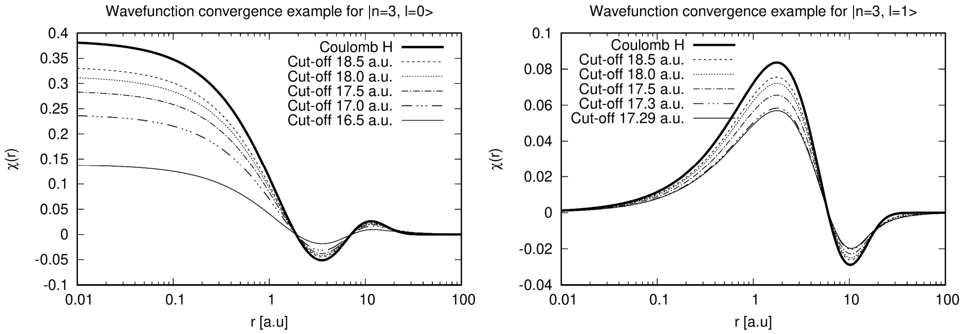

Figure 2: Left figure, the plasma influence onto the form of hydrogen wave function, n = 3, l = 0. Please note that $\chi(r) = r\,R(r)$, where $R(r)$ is a radial part of wave function. Right, the plasma influence onto the form of hydrogen wave function, n = 3, l = 1.

The dipole moment is used for the calculation of all of the properties of dense plasma. For instance, as a illustration, the total absorption cross section is proportional to the dipole matrix element

$$\left(\sigma_0\right)_{\omega=\omega_{fi}} = \frac{1}{3}\frac{g_2}{g_1}\frac{\pi\omega_{fi}}{\epsilon_0 \hbar c}\left|D\left(r;r_c;n_i,l_i;n_f,l_f\right)\right|^2, \tag{2}$$

and so any change of the wave function reflects on both dipole matrix element as well as total cross section symmetrically.

Table 1: The example set of dipole moments for hydrogen atom without plasma influence used for generation of behavior graphs on Figure 3.

Transition	$\lvert D\rvert^2$ [a_0]
|n=2,l=1> → |n=1,l=0>	1.29027
|n=3,l=1> → |n=2,l=0>	3.06482
|n=3,l=1> → |n=1,l=0>	0.516689
|n=3,l=0> → |n=2,l=1>	0.938404
|n=3,l=2> → |n=2,l=1>	4.74799
|n=9,l=1> → |n=1,l=0>	0.0820451
|n=9,l=1> → |n=2,l=0>	0.265475
|n=9,l=0> → |n=2,l=1>	0.0771136
|n=9,l=2> → |n=2,l=1>	0.314406

The test set of dipole matrix elements for the hydrogen atom in plasma calculated for the variety of cut-off radius, r_c, values is available at *https://github.com/nsakan972/ESPM-16.git.*

In order to have a correct modeling the influence of plasma microfield should be estimated. Within the presented model a cut-off radius r_c is used to model the plasma microfield. The investigation of the bound-bound transitions, i.e. the photoexcitation, is performed here.

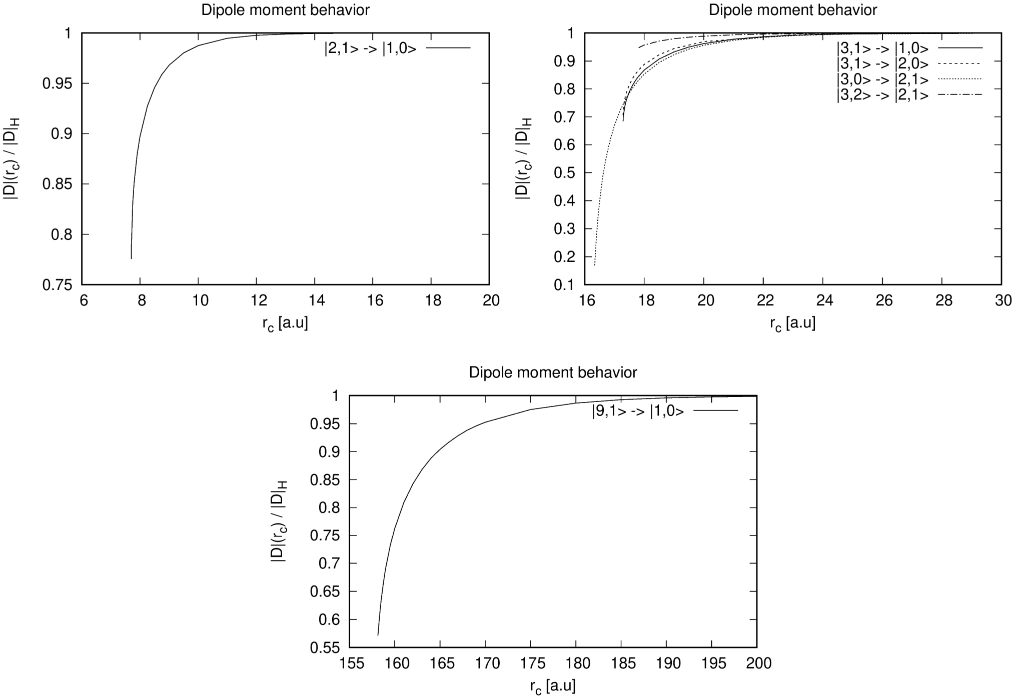

Figure 3: The behavior of normalized dipole moment regarding plasma influence for the transition |n=2,l=1> → |n=1,l=0>, left top graph. Plasma influence for the transition from upper levels with major quantum number n=3, right top graph. And plasma influence for the transition from principal quantum number n=9 to n=1, middle bottom graph.

As it could be seen from figures on Figure 3., the influence of the plasma micro field diminishes when the upper level is deep within the bound states energies, e.g. when it is burred deep within the bound states energies. When the upper level reaches the close vicinity of the boundary of the bound level states, e.g. if the realized bound state energy is close to zero in a potential given by equation (1), the plasma influence is no more neglectable.

It could be seen that the form of the dipole moment behavior is simple. It is a continuous and smooth function, so by knowing a cut-off radius at which the upper bound level starts to appear as well as a set of parameters for appropriate analytical function, a fast calculation of all related plasma parameters could be carried out. This gives an opportunity to perform a more complex modeling of a plasma screening boundary, at the vicinity of the model cut-off parameter r_c, and by such to ease the inclusion of more complex interactions, previously neglected by the model potential.

4. CONCLUSION

The presented work is a continuation of the previously developed modeling for the photoionization and inverse Bremsstrahlung processes of hydrogen plasma, and the goal is to include bound-bound processes within the same model. All of previously calculated data is also usable in modeling of Solar plasma processes.

In order to model a behavior of the plasma optical characteristics the used potential is a good approximation for the modeling of plasma interaction in a large area of densities and temperatures, details in Sakan et al. (2018). A strong plasma influence onto the optical parameters is observed where the plasma interaction energy is close to observed level energy value, e,g. when this level starts to appear, as illustrated in figures and analyzed within the manuscript. The work on using a more complex potentials is going on. We have tested a numerical method of wave function solution, and as a first step the Ar atom modeled is introduced, and initial values are in a expected range. As a second effect, a more detailed plasma-emitter interaction could be modeled. For both experimental praxis as well as Sun processes modeling an introduction of He atom and ion model is must, and as such could determine the further research.

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