Fluctuating governing parameters in galaxy dynamo

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Introduction

- Some galaxies have magnetic fields of several μG.
- Their evolution is described by the so-called dynamo theory (Beck et al., 1996).
- It is important to describe the magnetic field in galaxies with such processes as star formation, supernova explosions, outflows from stars etc.

Dynamo mechanism

- The dynamo mechanism is based on joint action of alpha-effect and differential rotation.
- Alpha-effect transforms the angular component of the field to the radial one:

$$B_\varphi \xrightarrow{\alpha} B_r$$

- Differential rotation transforms the radial field to angular:

$$B_\varphi \xrightarrow{\Omega} B_r$$

Basic equation

- The field is described by Steenbeck – Krause – Rädler equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}\left[\mathbf{v}, \mathbf{B}\right] + \mathrm{curl}\left(\alpha \mathbf{B}\right) + \eta \Delta \mathbf{B}.$$

no-z approximation

- The galaxy disc is very thin.
- We can neglect the z-component of the field.
- The z-derivatives of the magnetic field can be replaced by algebraic expressions (Moss, 1995; Phillips, 2001):

$$\frac{\partial^2 B_{r,\varphi}}{\partial z^2} \approx -\frac{\pi^2}{4h^2} B_{r,\varphi}.$$

System of equations

- The equations will be rewritten as:

$$\frac{\partial B_r}{\partial t} = -\frac{\alpha}{h} B_\varphi - \eta \frac{\pi^2}{4h^2} B_r + \eta \Delta_r B_r;$$

$$\frac{\partial B_\varphi}{\partial t} = r \frac{\partial \Omega}{\partial r} B_r - \eta \frac{\pi^2}{4h^2} B_\varphi + \eta \Delta_r B_\varphi.$$

- h is the half-thickness of the disc, α is the alpha-effect coefficient, $\eta = lv/3$ is the turbulent diffusivity coefficient.

Galaxies with active processes

- Usually averaged values of the coefficients are used.
- This approach is useful for “calm” galaxies, where the kinematical characteristics are nearly the same in different parts of galaxies.
- If there are active star formation, supernova explosions etc, the kinematical characteristics can differ very much.

Random coefficients

- The HII regions usually exist for quite small times (about 10^7 years). Their location can be described by random laws.
- Some works described the dynamo model with random alpha-effect (Mikhailov & Modyaev, 2015) and with random injections of the magnetic fields (Moss et al., 2015).

Diffusivity

- We describe the equations with random diffusivity coefficient.
- For “calm” regions $v_0=10$ km/s, for HII regions $v_1=30$ km/s.
- So for the diffusivity coefficient $\eta=lv/3$ we have:

$$\eta_1=3\eta_0$$

Dimensionless form

- We can rewrite the equations, measuring time in h^2/η_0, and neglect the diffusion in disc plane:

$$\frac{dB_r}{dt} = -R_\alpha B_\varphi - kB_r;$$

$$\frac{dB_\varphi}{dt} = R_\omega B_r - kB_\varphi.$$

- R_α characterizes alpha-effect, R_ω characterizes differential rotation, k characterizes turbulent diffusion.

k coefficient

- The k coefficient takes random values:

$$k = \begin{cases} 7.5 \text{ with probability } p; \\ 2.5 \text{ with probability } (1-p). \end{cases}$$

- The coefficient is constant for $\Delta t = 0.01$ and after that renews.
- p characterizes the intensity of active processes.

Theoretical approximations

- The field grows exponentially with velocity:

$$\gamma = -k \pm \sqrt{R_\alpha R_\omega}.$$

- The main features are described by the highest value.
- If we take $R_\alpha=1$, $R_\omega=10$, the values of velocities for k_0 and k_1 will be:

$$\gamma_0=0.66,\ \gamma_1=-4.33$$

Magnetic field for large time

- The magnetic field will be:

$$B(n\Delta t)=B(0)\exp(\gamma(0)\Delta t)\ldots\exp(\gamma((n\text{-}1)\Delta t)\Delta t)$$

- With probability $\binom{m}{n} p^m (1-p)^{n-m}$ the field is:

$$B(n\Delta t)=B(0)\exp(m\gamma_1\Delta t)\exp((n\text{-}m)\gamma_0\Delta t)$$

- Using these values, we can average the field and its square.

Different momentums

- Mean value:

$$\langle B(n\Delta t)\rangle = B(0)\exp(\gamma_0 n\Delta t)(1-p+p\exp((\gamma_1-\gamma_0)\Delta t))^n$$

- Mean-square field:

$$\langle B^2(n\Delta t)\rangle^{1/2} = B(0)\exp(\gamma_0 n\Delta t)(1-p+p\exp(2(\gamma_1-\gamma_0)\Delta t))^{n/2}$$

Estimeates for velocities of different momentums

- Typical realization:

$$\lambda_0 = 0.66 - 5p$$

- Mean field:

$$\lambda_1 = 0.66 - 5p + 12.5p(1-p)\Delta t$$

- Mean-square field:

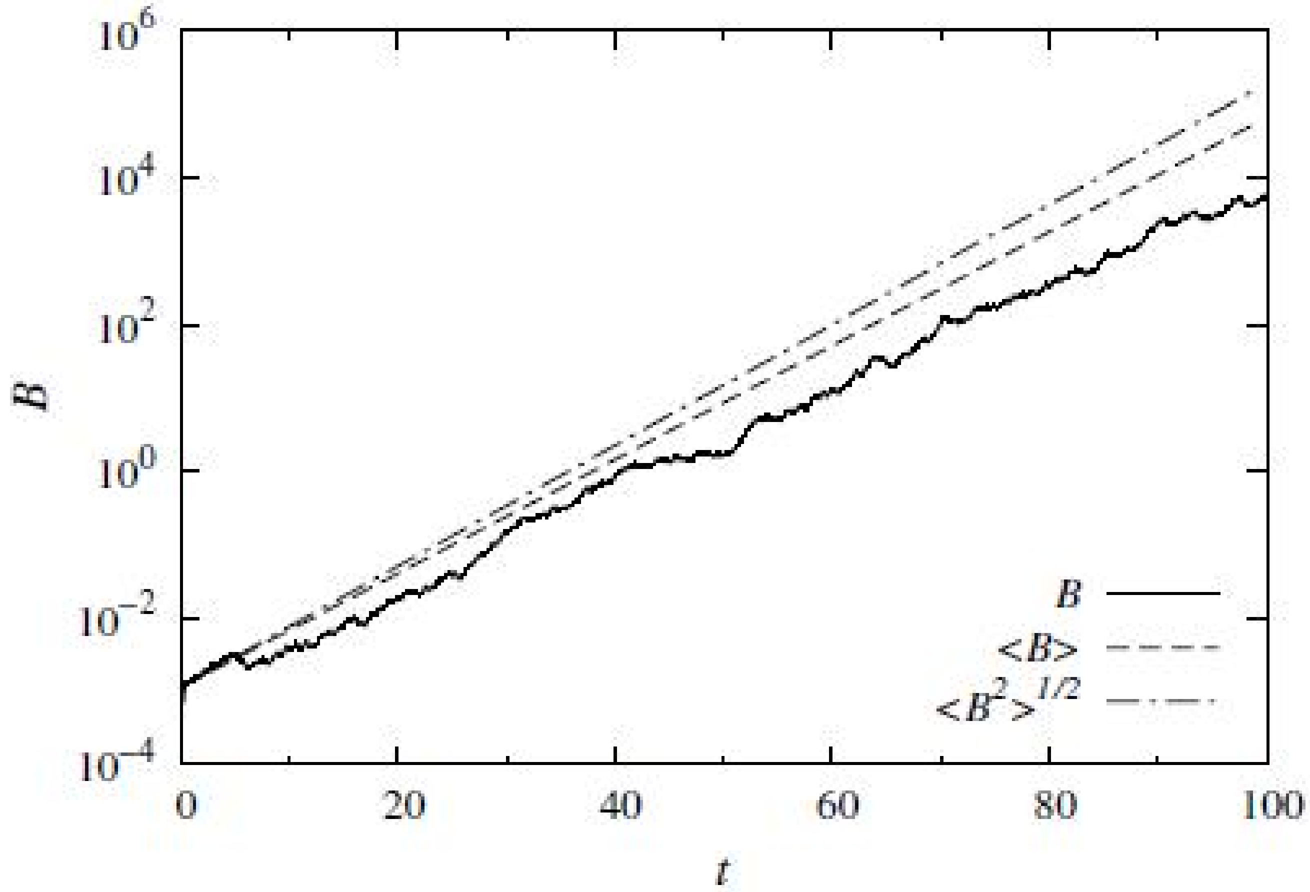
$$\lambda_2 = 0.66 - 5p + 25p(1-p)\Delta t$$

- Higher momentums grow faster than lower ones – intermittency effect.

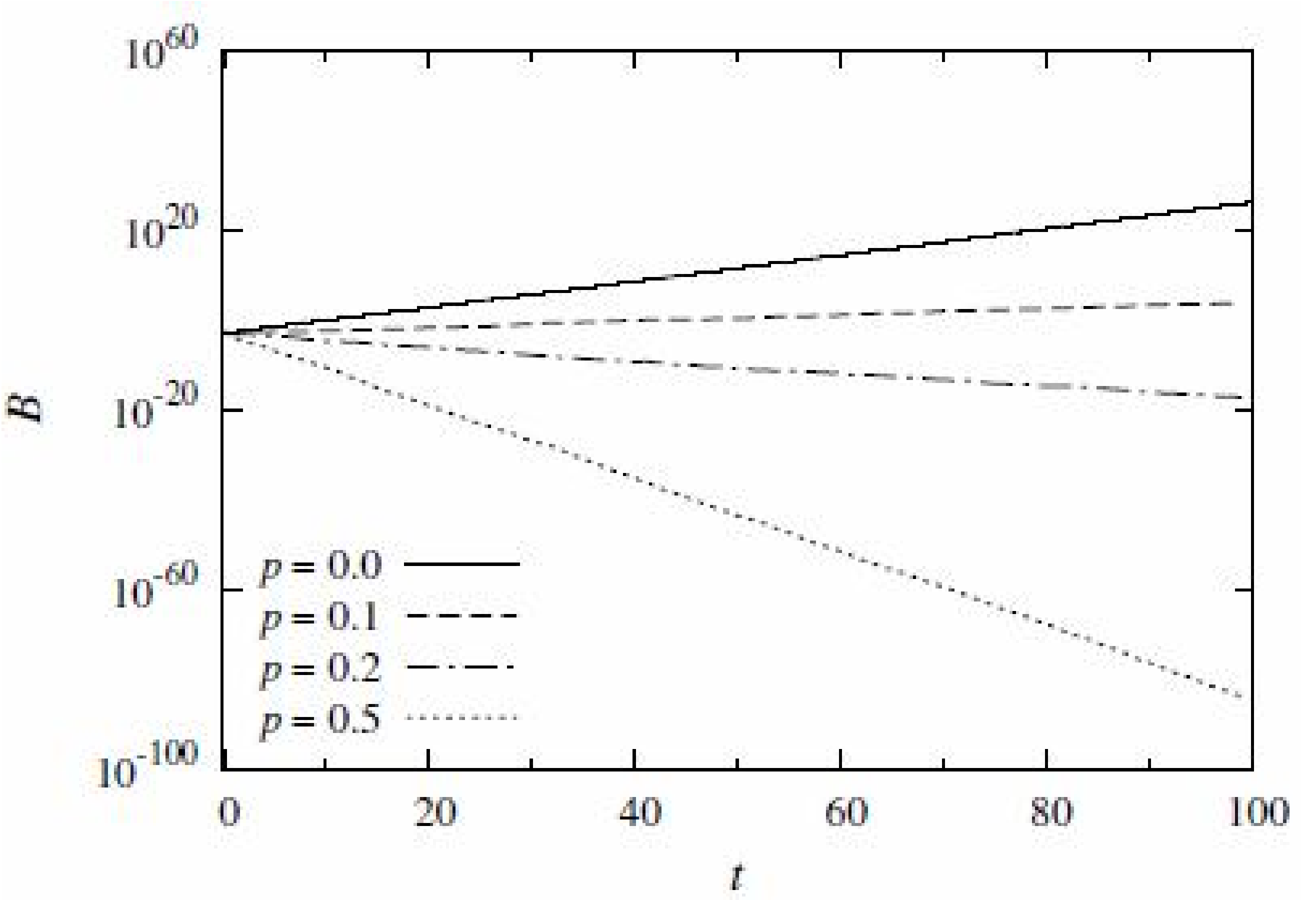
Critical value (theoretical estimates)

- Assuming $\lambda_0=0$, we will obtain $p_{cr}=0.13$.
- $\lambda_0>0$ if $p>0.13$ (the field grows).
- $\lambda_0<0$ if $p>0.13$ (the field decays).

Numerical results (p=0.1)



Various probabilities



Growth rates

	Numerical values			Theoretical estimates		
p	λ_0	λ_1	λ_2	λ_0	λ_1	λ_2
0	0.671	0.680	0.681	0.662	0.662	0.662
0.1	0.182	0.180	0.181	0.162	0.174	0.185
0.2	-0.366	-0.300	-0.280	-0.338	-0.318	-0.298
0.5	-1.86	-1.79	-1.76	-1.84	-1.81	-1.78

Critical value (numerical)

- If $p<0.16$, the field grows.
- If $p>0.16$, it decays.
- The critical value ($p_{cr}=0.16$) is higher than for rough estimates.

Star formation and our model

- The probability p can be associated with the fraction κ of the HII regions in the galaxy. For rough approximation $\kappa \approx p$.
- If we study the star formation, it can be shown (e.g. Mikhailov, 2014), that

$$\kappa \approx 12 \Sigma_{SFR}$$

(star formation density measured in $M_{\odot}$/yr kpc^2)

Growth rates – in dimensional form

	Numerical values			Theoretical estimates		
Σ_{SFR}, $M_{\odot}$/yr kpc^2	λ_{0}, Gyr^{-1}	λ_{1}, Gyr^{-1}	λ_{2}, Gyr^{-1}	λ_{0}, Gyr^{-1}	λ_{1}, Gyr^{-1}	λ_{2}, Gyr^{-1}
0	0.906	0.919	0.920	0.895	0.895	0.895
0.012	0.246	0.243	0.245	0.219	0.235	0.250
0.024	-0.495	-0.405	-0.378	-0.457	-0.429	-0.402
0.06	-2.51	-2.42	-2.38	-2.49	-2.46	-2.41

Critical value for star formation

- For star formation we can obtain $\Sigma_{cr} \approx 0.013 M_{\odot}$/yr kpc^2.
- If $\Sigma_{SFR} > \Sigma_{cr,}$ the field decays.
- This value is few times more than in the Milky Way.

Summary

- The dynamo model with random diffusivity coefficients has been studied. It can be useful to describe the influence of active processes on the magnetic field.
- According to this approach, intensive active processes make the field decay. As for the star formation rate, the field decays if $\Sigma_{SFR} > \Sigma_{cr} \approx 0.013 M_{\odot}$/yr kpc^2.
- It is quite similar to the results obtained in deterministic dynamo model with star formation (Mikhailov et al., 2012; Mikhailov, 2014).
- It is interesting to study a model where both alpha-effect coefficient and the diffusion coefficient are random. It can make our results more precise.

References

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THANK YOU FOR ATTENTION!