R^n gravity as viable alternative to dark matter: application to stellar dynamics

V. Borka Jovanović[1], P. Jovanović[2], D. Borka[1], S. Capozziello[3,4]

[1]Atomic Physics Laboratory (040), Vinča Institute of Nuclear Sciences, P.O. Box 522, 11001 **Belgrade, Serbia**
[2]Astronomical Observatory, Volgina 7, P.O. Box 74, 11060 **Belgrade, Serbia**
[3]Dipartimento di Fisica, Università di Napoli "Federico II",
[4]INFN - Sezione di Napoli, Via Cinthia, **Napoli, Italy**

Outline of the talk

- Object and goal: to present R^n gravity application to stellar dynamics around Galactic Center

- Motivation: the role of $f(R)$ gravity, as well as the other modifications of standard Einstein's gravity, is to explain some observed phenomena without dark matter and dark energy

- Data (NTT/VLT and Keck telescope):
  Gillessen et al. 2009, ApJ, 692, 1075; Ghez et al. 2008, Apj, 689, 1044

- Method: R^n is analyzed using observed orbits of S-stars and also using two-body simulations
- Results: we review the various consequences of the $f(R)$ gravity parameters on stellar dynamics and investigate their constraints from the observed S-star orbits

- Conclusions
- References

Extended Theories of Gravity (ETGs)

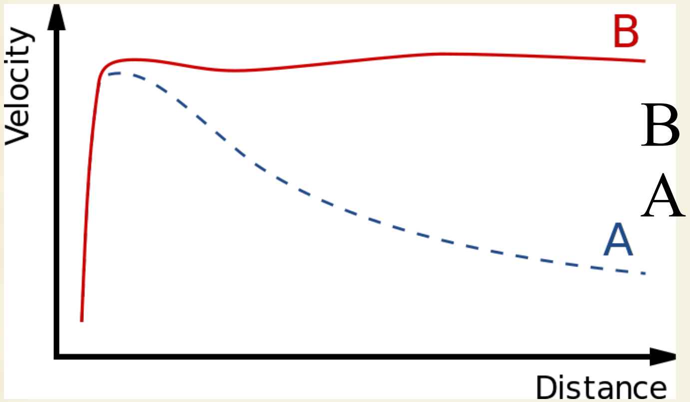
ETGs are physical theories that attempt to describe the phenomena of gravitation in competition to Einstein's theory of general relativity, by preserving the undoubtedly positive results of Einstein's theory.

Instead of introducing Dark Matter (DM), which is a hypothetical type of matter does not emitting or interacting with electromagnetic radiation, some theories, which modify the laws of gravity, could explain in a very natural fashion several astrophysical and cosmological observations.

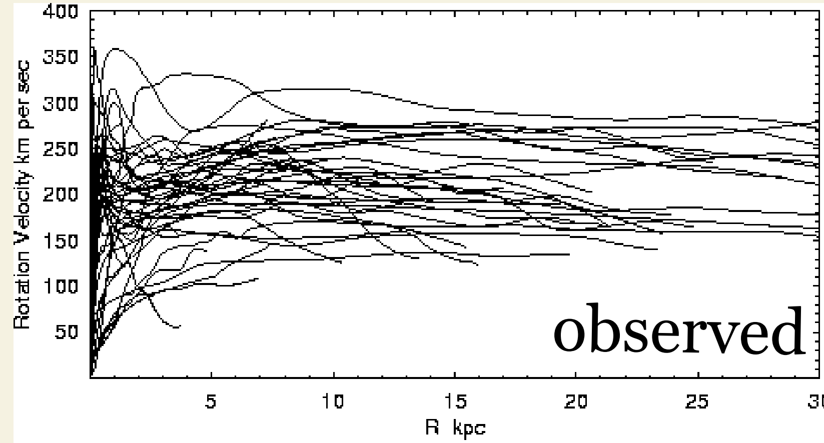
- it is aimed to address conceptual and experimental problems recently emerged in astrophysics, cosmology and high energy physics (such as rotational velocity curves of spiral galaxies, their baryonic Tully-Fischer relation and fundamental plane of elliptical galaxies)

- some of the alternatives to general relativity:
  MOND: (Milgrom 1983)
  ETG: (f(R) gravity (family of models f(R,φ), f(T), Yukawa-like …),
  Brans-Dicke gravity, scalar-tensor theories, Palatini formalism,
  conformal transformations ...

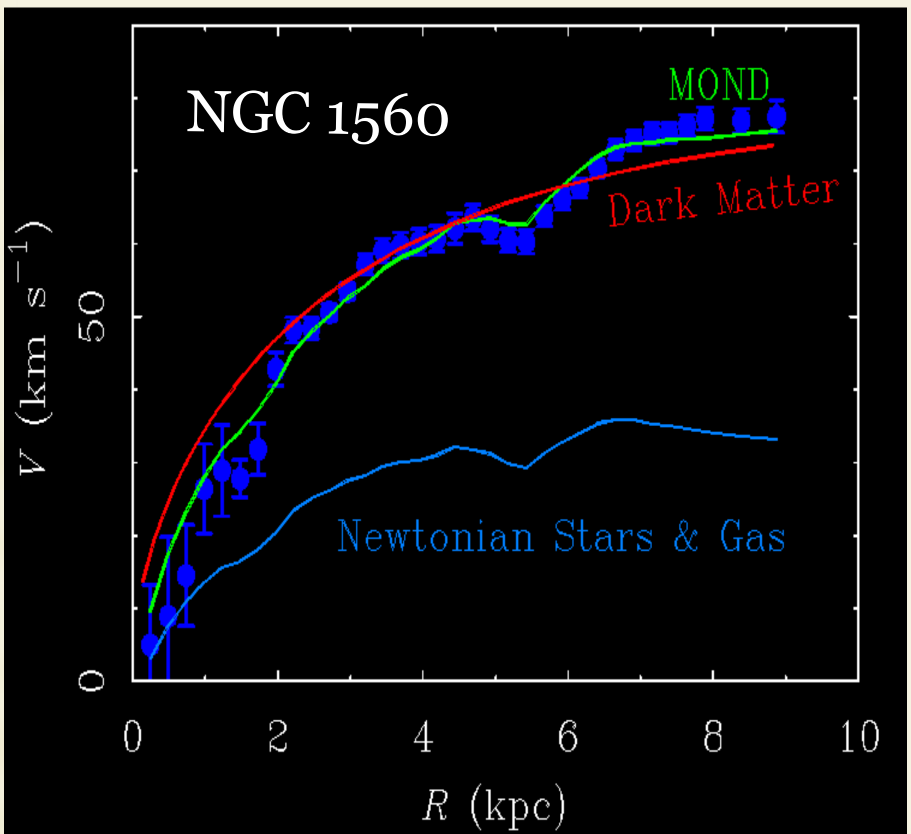
Modified gravity and rotation curves of spiral galaxies

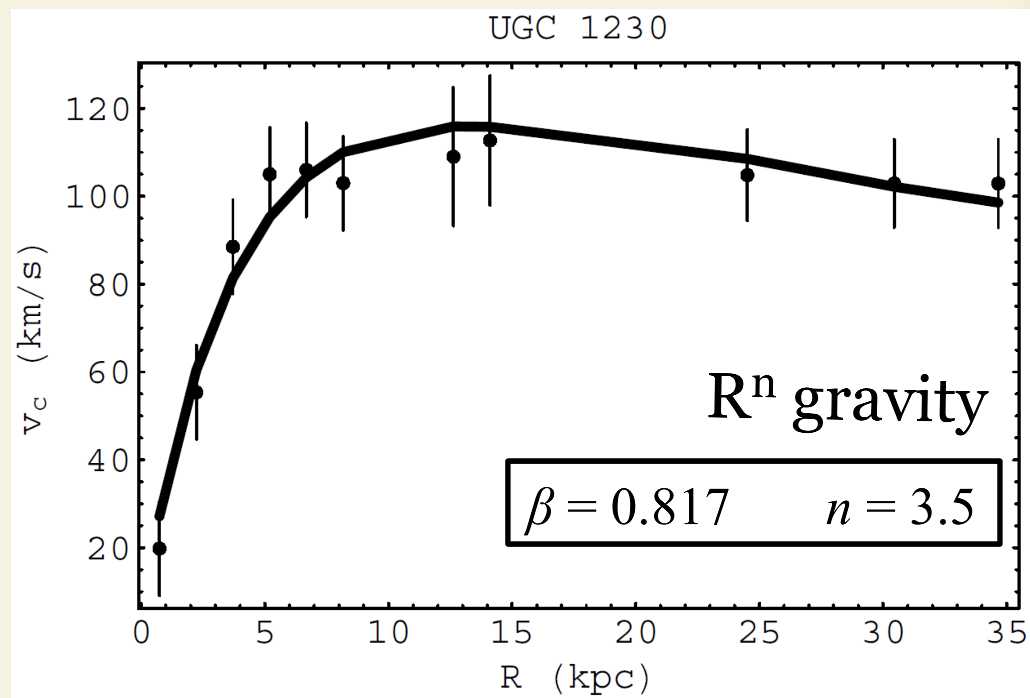


B - observation
A - Newtonian prediction



Sofue & Rubin, 2001, ARA&A, 39, 137





Capozziello et al. 2007, MNRAS, 375, 1423

Basic theory I - *f*(R) modified gravity

The role of *f*(R) gravity, as well as the other modifications of standard Einstein's gravity, is to explain the accelerated expansion, structure formation of the Universe, and some other phenomena at extragalactic scales (such as e.g. flat rotation curves of spiral galaxies) without adding unknown forms of dark energy or dark matter.

See review in: S. Capozziello, M. De Laurentis, *Extended Theories of Gravity*, Phys. Rep. 509, 167-321 (2011).

8.2.2. $f(R)$-gravity: The general case

Let us discuss now a generic analytical[15] function $f(R)$ in the metric formalism, beginning with the vacuum case, as described by the Lagrangian density $\sqrt{-g}\,\mathcal{L} = \sqrt{-g}\,f(R)$ obeying the variational principle $\delta\int d^4x\sqrt{-g}\,f(R) = 0$. We have

$$\delta\int d^4x\sqrt{-g}\,f(R) = \int d^4x\left[\delta\left(\sqrt{-g}\,f(R)\right) + \sqrt{-g}\,\delta\left(f(R)\right)\right]$$
$$= \int d^4x\sqrt{-g}\left[f'(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R)\right]\delta g^{\mu\nu} + \int d^4x\sqrt{-g}\,f'(R)g^{\mu\nu}\delta R_{\mu\nu}, \tag{8.26}$$

where the prime denotes differentiation with respect to R. We now compute these integrals in the local inertial frame. By using

$$g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}\partial_\sigma\left(\delta G^\sigma_{\mu\nu}\right) - g^{\mu\sigma}\partial_\sigma\left(\delta G^\nu_{\mu\nu}\right) \equiv \partial_\sigma W^\sigma \tag{8.27}$$

where

$$W^\sigma \equiv g^{\mu\nu}\delta G^\sigma_{\mu\nu} - g^{\mu\sigma}\delta G^\nu_{\mu\nu}, \tag{8.28}$$

the second integral in Eq. (8.26) can be written as

$$\int d^4x\sqrt{-g}\,f'(R)g^{\mu\nu}\delta R_{\mu\nu} = \int d^4x\sqrt{-g}\,f'(R)\partial_\sigma W^\sigma. \tag{8.29}$$

Integration by parts yields

$$\int d^4x\sqrt{-g}\,f'(R)g^{\mu\nu}\delta R_{\mu\nu} = \int d^4x\frac{\partial}{\partial x^\sigma}\left[\sqrt{-g}\,f'(R)W^\sigma\right] - \int d^4x\partial_\sigma\left[\sqrt{-g}f'(R)\right]W^\sigma. \tag{8.30}$$

The first integrand is a total divergence and can be discarded by assuming that the fields vanish at infinity, obtaining

$$\int d^4x\sqrt{-g}\,f'(R)g^{\mu\nu}\delta R_{\mu\nu} = -\int d^4x\partial_\sigma\left[\sqrt{-g}\,f'(R)\right]W^\sigma. \tag{8.31}$$

Let us calculate now the term W^σ appearing in Eq. (8.31). We have

$$\delta G^\sigma_{\mu\nu} = \delta\left[\frac{1}{2}g^{\sigma\alpha}\left(\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu}\right)\right] = \frac{1}{2}g^{\sigma\alpha}\left[\partial_\mu\left(\delta g_{\alpha\nu}\right) + \partial_\nu\left(\delta g_{\mu\alpha}\right) - \partial_\alpha\left(\delta g_{\mu\nu}\right)\right], \tag{8.32}$$

since in the locally inertial frame considered here it is

$$\partial_\alpha g_{\mu\nu} = \nabla_\alpha g_{\mu\nu} = 0. \tag{8.33}$$

Similarly, it is

$$\delta G^\nu_{\mu\nu} = \frac{1}{2}g^{\nu\alpha}\partial_\mu\left(\delta g_{\nu\alpha}\right). \tag{8.34}$$

By combining Eqs. (8.33) and (8.34), one obtains

$$g^{\mu\nu}\delta G^\sigma_{\mu\nu} = \frac{1}{2}g^{\mu\nu}\left[-\partial_\mu\left(g_{\alpha\nu}\delta g^{\alpha\sigma}\right) - \partial_\nu\left(g_{\mu\alpha}\delta g^{\sigma\alpha}\right) - g^{\sigma\alpha}\partial_\alpha\left(\delta g_{\mu\nu}\right)\right]$$
$$= \frac{1}{2}\partial^\sigma\left(g_{\mu\nu}\delta g^{\mu\nu}\right) - \partial^\mu\left(g_{\alpha\mu}\delta g^{\nu\alpha}\right), \tag{8.35}$$

$$g^{\mu\sigma}\delta G^\nu_{\mu\nu} = -\frac{1}{2}\partial^\sigma\left(g_{\nu\alpha}\delta g^{\nu\alpha}\right), \tag{8.36}$$

from which it follows immediately that

$$W^\sigma = \partial^\sigma\left(g_{\mu\nu}\delta g^{\mu\nu}\right) - \partial^\mu\left(g_{\mu\nu}\delta g^{\sigma\nu}\right). \tag{8.37}$$

Using this equation one can write

$$\int d^4x\sqrt{-g}\,f'(R)g^{\mu\nu}\delta R_{\mu\nu} = \int d^4x\partial_\sigma\left[\sqrt{-g}\,f'(R)\right]\left[\partial^\mu\left(g_{\mu\nu}\delta g^{\sigma\nu}\right) - \partial^\sigma\left(g_{\mu\nu}\delta g^{\mu\nu}\right)\right]. \tag{8.38}$$

15 This assumption is not, strictly speaking, necessary and is sometimes relaxed in the literature.

Basic theory II - R^n modified gravity

- we adopt *f*(R) gravity which is the straightforward generalization of Einstein's General Relativity as soon as the function is $f(R) \neq R$, that is, it is not linear in the Ricci scalar R as in the Hilbert-Einstein action

See, for example, consideration of the power-law fourth-order theories of gravity in S. Capozziello, V. F. Cardone, A. Troisi, *Low surface brightness galaxy rotation curves in the low energy limit of R^n gravity: no need for dark matter?*, Mon. Not. R. Astron. Soc. 375, 1423-1440 (2007).

R^n gravity is the power-law version of $f(R)$ modified gravity. In the weak field limit, its potential is:

$$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right]$$

r_c - scalelength depending on the gravitating system properties
β - universal constant

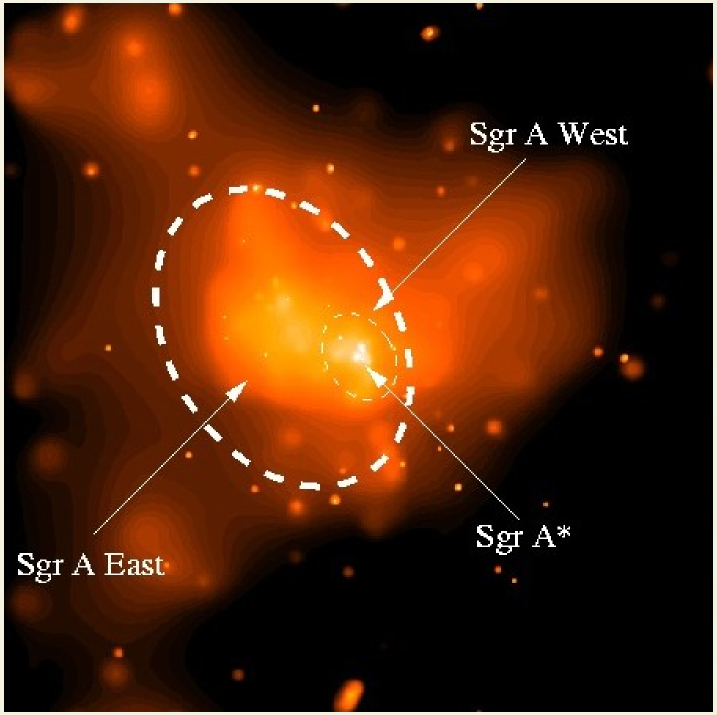
$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}$$

$n = 1 \rightarrow \beta = 0$
Newtonian case

Sagittarius A and S-stars

Sagittarius A (or Sgr A) is a complex radio source that consists of three components, which overlap:

(1) Sgr A East (the supernova remnant)
(2) Sgr A West (the spiral structure)
(3) Sgr A* (a very bright compact radio source at the center of the spiral)



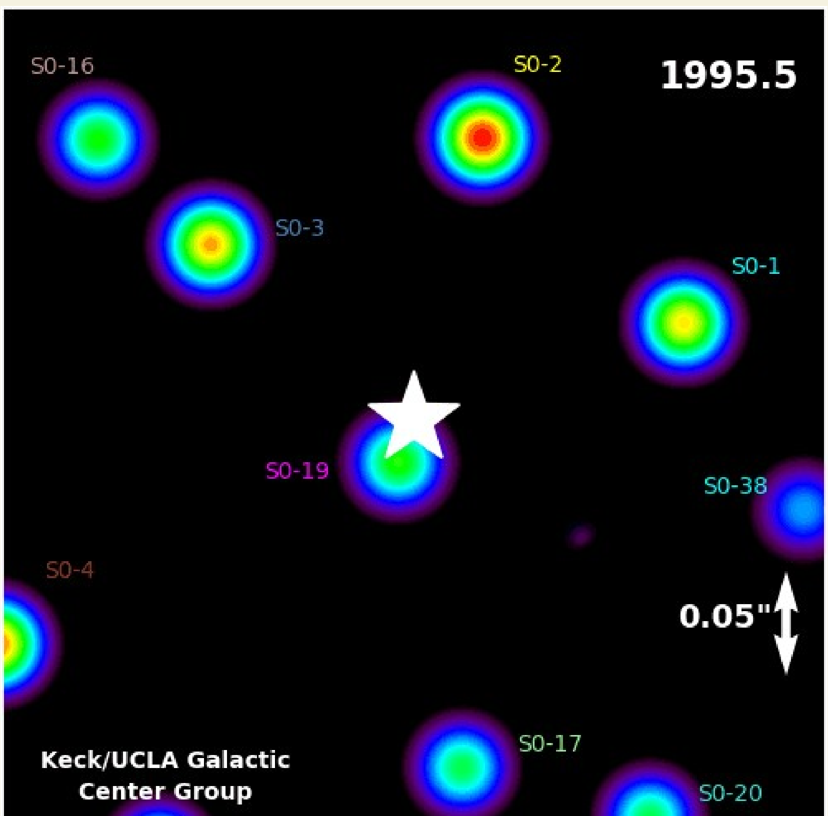
Chandra X-ray image (NASA/ Penn State/ G. Garmire et al.)

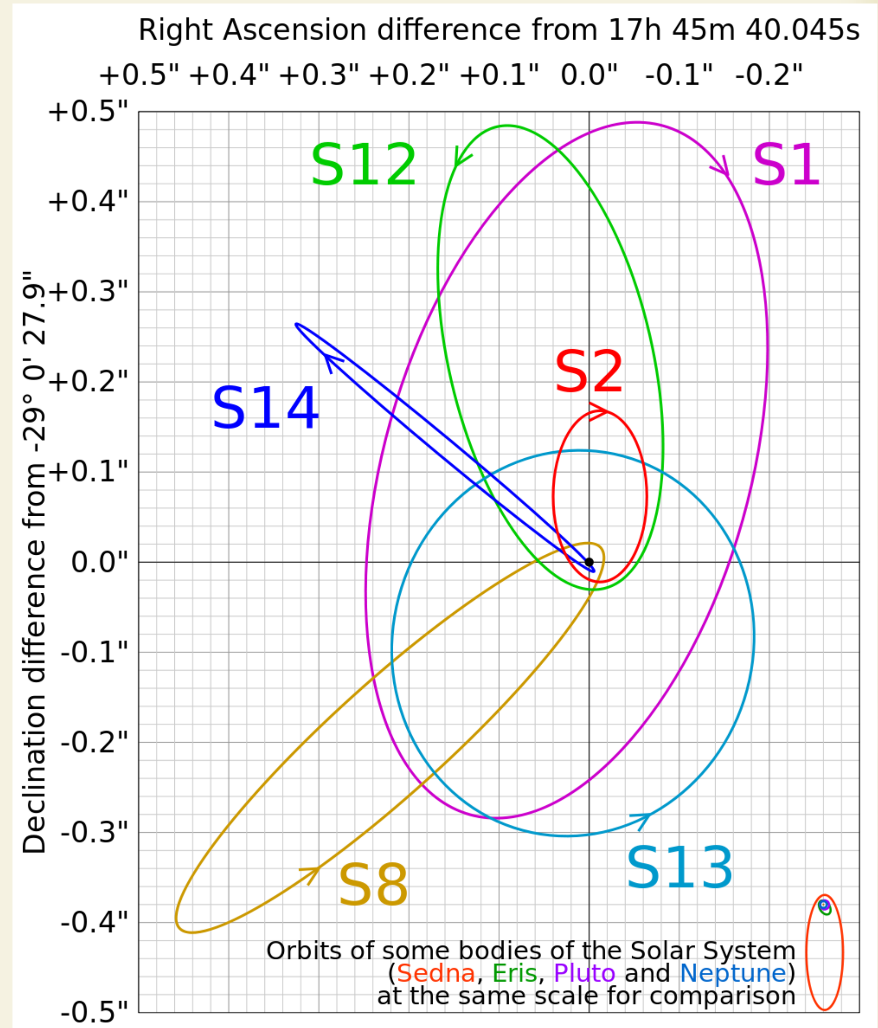
Sgr A* is very compact and motionless source, and its location coincides with the dynamical center of the Galaxy. The massive black hole Sgr A* at the Galactic center is surrounded by a cluster of stars orbiting around it: S-star cluster. Light from these stars is bent by the gravitational field of the black hole.

S2 star

S-stars are orbiting with large velocities (v > 1000 km/s), and have very eccentric orbits around central SMBH at GC.

S2 star is one of the brightest members of the S-star cluster. It has about 15 Solar masses and seven times its diameter, with orbital period about 15.8 yr.





Projection on the sky of some S star orbits (Eisenhauer et al. 2005).

Observations and method

Observational data are publicly available as the supplementary online data to the electronic version of paper S. Gillessen, F. Eisenhauer, T. K. Fritz, H. Bartko, K. Dodds-Eden, O. Pfuhl, T. Ott, R. Genzel, Astrophys. J. 707, L114 (2009): NTT/VLT and Keck optical telescopes.

New Technology Telescope (3.6 m), *La Silla Obs., Chile*

Very Large Telescope (4 x 8.2 m), *Paranal Observatory, Chile*

Keck telescope (2 x 10 m), *Keck Observatory, Hawaii, USA*

- we performed two-body simulations in the R^n gravity potential
- we compare the obtained theoretical results for S2-like star orbits in the R^n potential with these two independent sets of observations of the S2 star

S2 star orbits in R^n and Newtonian potential

We draw orbits of S2 star in R^n and Newtonian potential.
Modified: $\Phi_R(x,y) = -\,GM/(2r)(1+(r/r_\mathrm{c})^\beta)$; Newton: $\Phi_N(x,y) = -\,GM/r$

β - parameter which depends on power n, we have:

$n \rightarrow \infty \Rightarrow \beta \rightarrow 1$

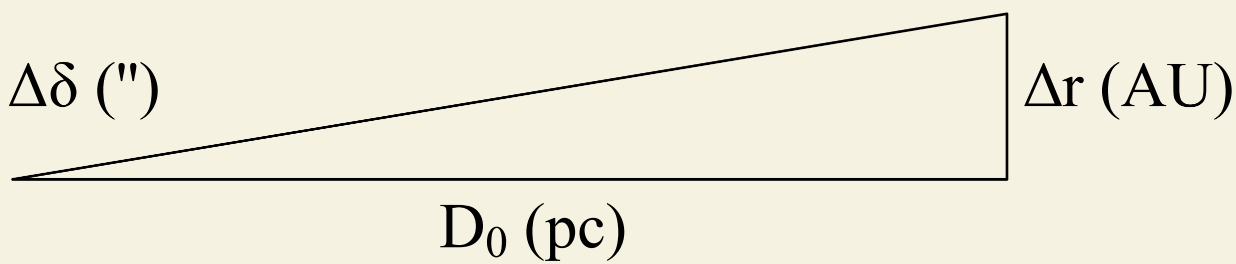
$n = 1 \Rightarrow \beta = 0$ (the gravitational field reduces to Newtonian)

r_c - arbitrary parameter (depends on observed system and its scale, for instance for the Sun as a source of gravitational field and the Earth as test particle, r_c is in the range 1-10^4 AU)

- for the same set of input parameters (t_0, x_0, y_0, v_0), we calculate Newtonian and modified (in this case R^n) trajectories and find a maximum distance between them
- for both potentials, for starting point we take the pericenter (point at the trajectory which is closest to the focus, i.e. black hole in Galactic Center)
- values of initial positions and velocities we choose in that way to obtain orbits with excentricity and period which correspond to S2 star
- we calculate Δr (in AU), and compare with ε (in ") multiplied with the distance.

Parameters β and r_c

- parameters should be chosen in the way that during the first period of moving, the trajectories should be as similar as possible (we have observations for 1T). We achieve this by giving some **maximal deviation ε** between modified and Newtonian trajectory.
- we search only for those orbits in R^n for which all deviations Δr, during the first orbiting period, are less than the certain value $\Delta r_{limit} = \varepsilon D_0$.



$1\ \text{pc} = 3.0857 \cdot 10^{16}\ \text{m}$
$= 206264.8\ \text{AU}$

D_0 - the distance between the observer and the binary system
(around that binary system S2 star is rotating) $D_0 = 8000$ pc

$\Delta\delta\ ('') = \Delta r\ (\text{AU}) / D_0\ (\text{pc})$

$\Delta r = \delta D_0$

$\boldsymbol{\Delta r_{limit} = \varepsilon D_0}$

$\varepsilon = 10^{-2}\,'' = 10^{-2}\ \text{AU/pc} \Rightarrow \Delta r_{limit} = 10^{-2}\ \text{AU/pc} \cdot 8000\ \text{pc} = 80\ \text{AU}$

Results I

Here we present an overview of some of our results.

PHYSICAL REVIEW D **85,** 124004 (2012)

Constraints on R^n gravity from precession of orbits of S2-like stars

D. Borka,[1,*] P. Jovanović,[2] V. Borka Jovanović,[1] and A. F. Zakharov[3,4]

In: Advances in General Relativity Research
Editor: Cameron Williams
ISBN: 978-1-63483-120-8

Chapter 9

S2 LIKE STAR ORBITS NEAR THE GALACTIC CENTER IN R^n AND YUKAWA GRAVITY

Duško Borka[1,], Predrag Jovanović[2], Vesna Borka Jovanović[1] and Alexander F. Zakharov[3,4,5,6,7]*

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Advances in Space Research 54 (2014) 1108–1112
ADVANCES IN SPACE RESEARCH
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Constraints on R^n gravity from precession of orbits of S2-like stars: A case of a bulk distribution of mass

A.F. Zakharov[a,b,c,d,e,*], D. Borka[f], V. Borka Jovanović[f], P. Jovanović[g]

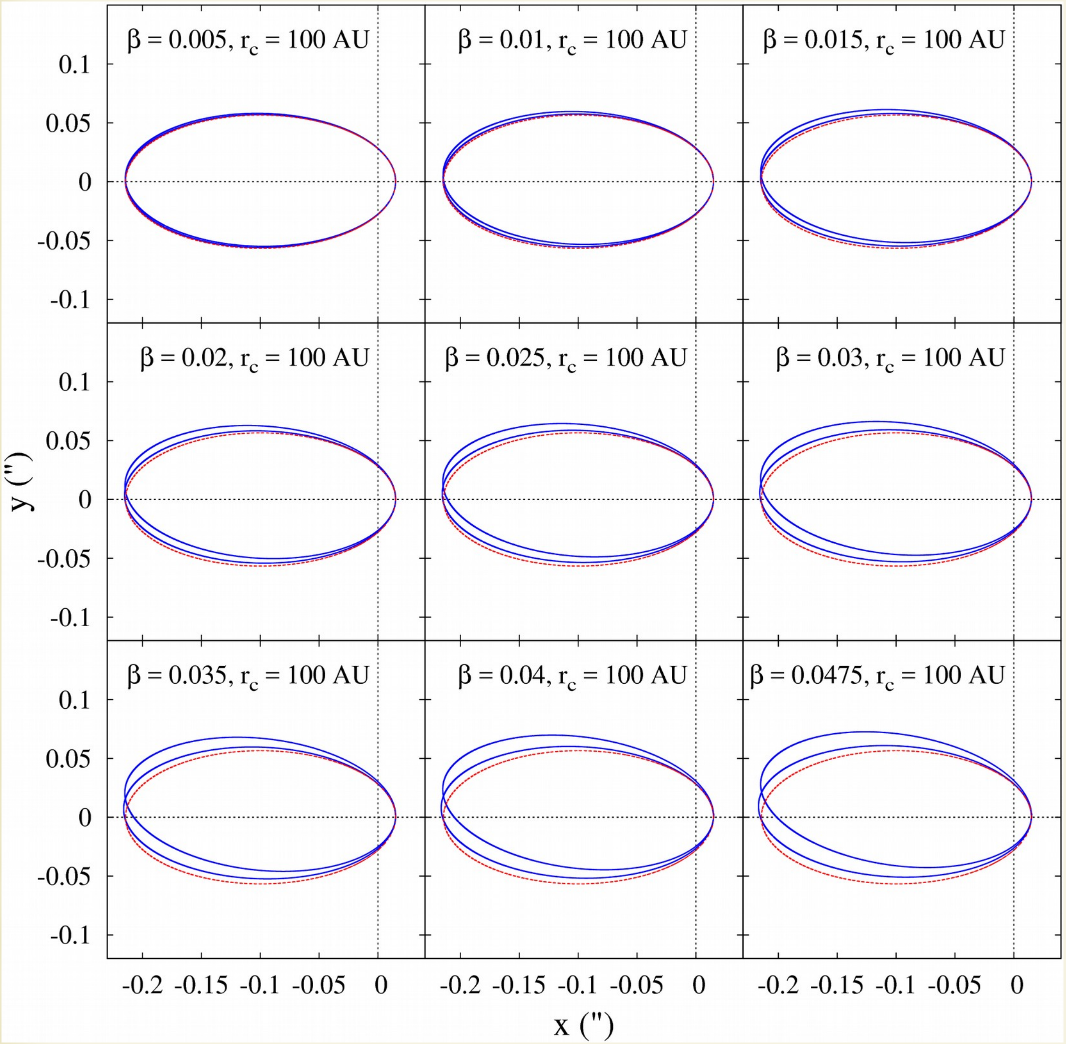
PHYSICAL REVIEW D **90,** 044052 (2014)

Constraining extended gravity models by S2 star orbits around the Galactic Centre

S. Capozziello,[1,2,3,*] D. Borka,[4] P. Jovanović,[5] and V. Borka Jovanović[4]

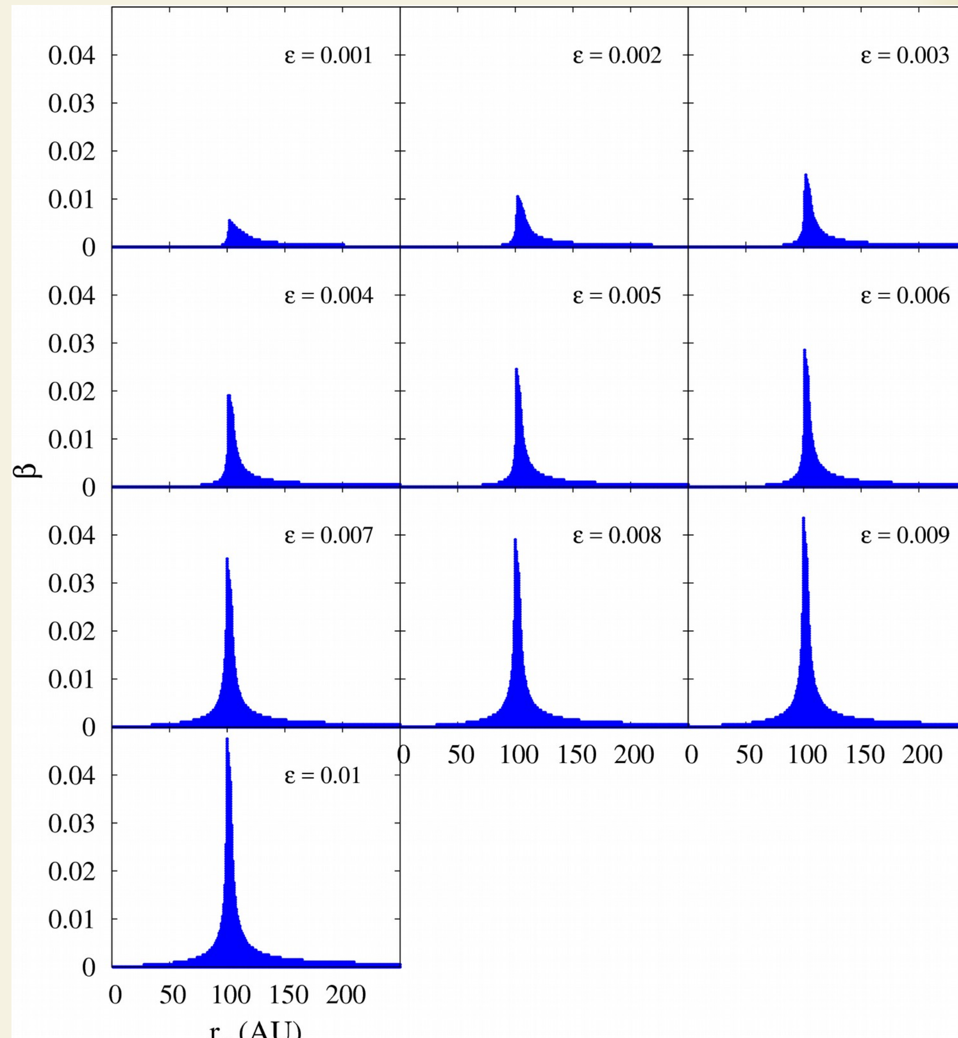
Results II

$\beta = 0.005\text{-}0.0475$



Orbits of S2-like stars for $r_c = 100$ AU.

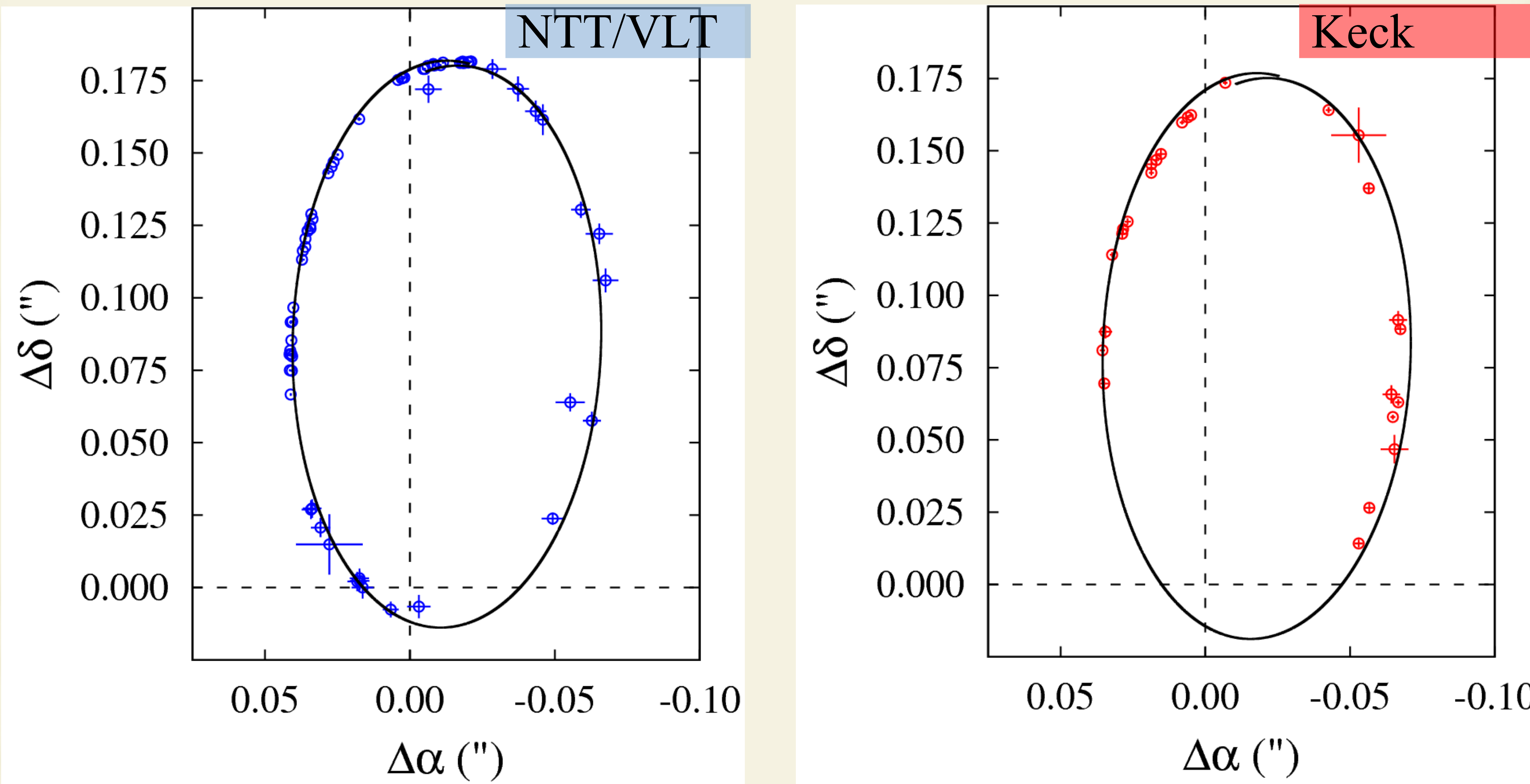
$\varepsilon = 0.001\text{-}0.01$



The parameter space.

Results III

$\beta = 0.01, \quad r_c = 100$ AU, $\quad \Delta\Theta \approx -1^{o}$ (orbital precession)



The comparison of the fitted orbit of S2 star and astrometric observations.

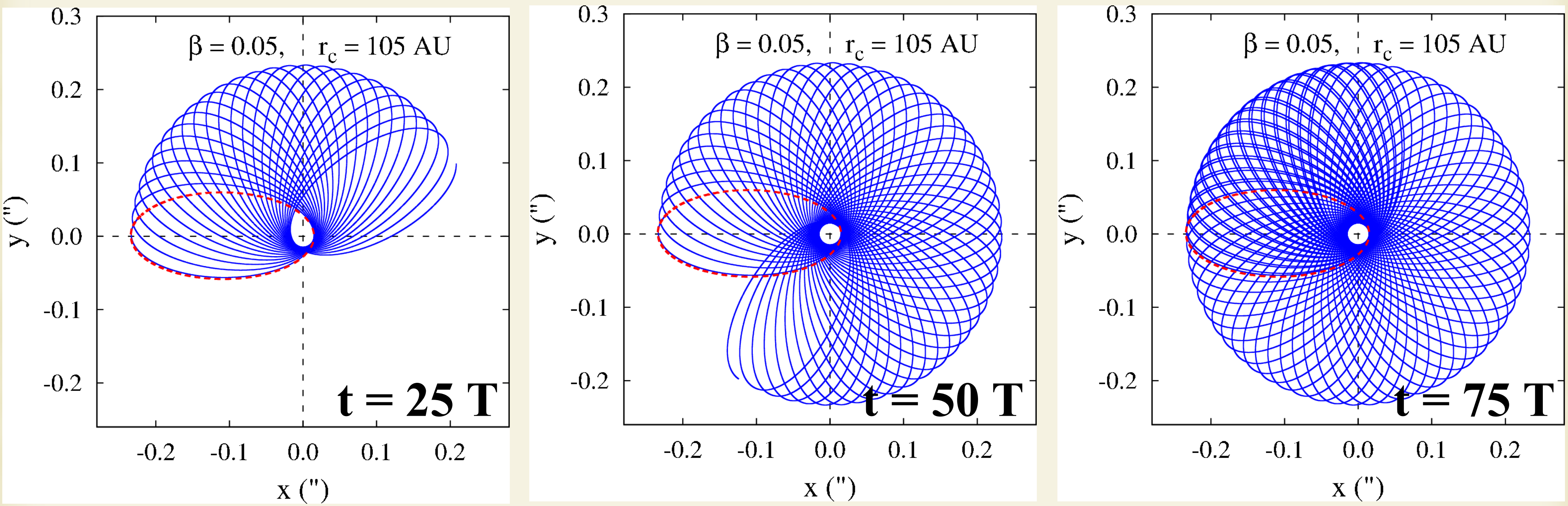
Results IV

orbital precession angle in R^n gravity:

$$\boxed{\begin{aligned}\Delta\theta &= \frac{\pi}{2}\beta(\beta-1)\left(\frac{a\left(1-e^2\right)}{r_c}\right)^{\beta}\times \\ &\times {}_2F_1\left(\frac{\beta+1}{2},\frac{\beta+2}{2};2;e^2\right),\end{aligned}}$$

$$\beta\approx 0 \qquad \Delta\theta = \frac{\pi^{rad}\beta\left(\sqrt{1-e^2}-1\right)}{e^2}$$

$$\beta\approx 1 \qquad \Delta\theta = \frac{\pi^{rad}a\left(\beta-1\right)\left(\sqrt{1-e^2}-1+e^2\right)}{r_c e^2}$$



R^n gravity has an effect similar to the extended mass distribution and produces a <u>retrograde</u> shift that results in rosette-shaped orbits.

Results V

- radial velocity in polar coordinates (r,θ):

$$v_{\rm rad} = \sin i[\sin(\theta + \omega) \cdot \dot{r} + r\cos(\theta + \omega) \cdot \dot{\theta}].$$

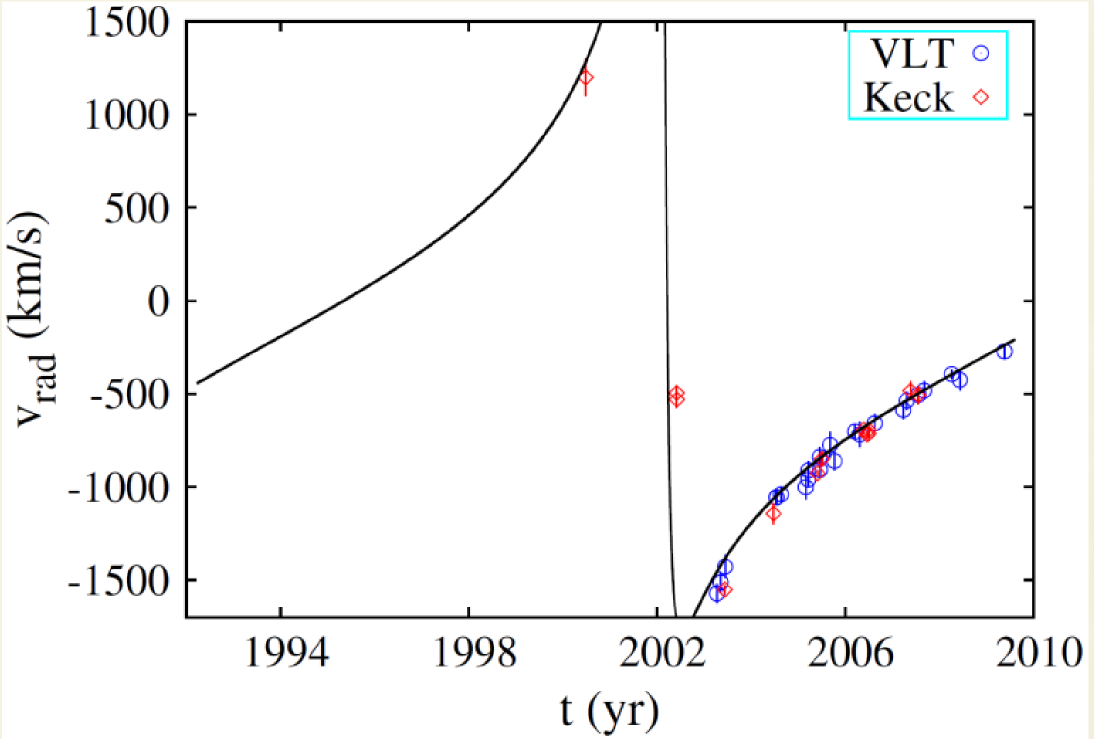
- we use rectangular coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

fitted radial velocities:

$$v_{\rm rad} = \frac{\sin i}{\sqrt{x^2 + y^2}}[\sin(\theta + \omega) \cdot (x\dot{x} + y\dot{y}) + \cos(\theta + \omega) \cdot (x\dot{y} - y\dot{x})],$$

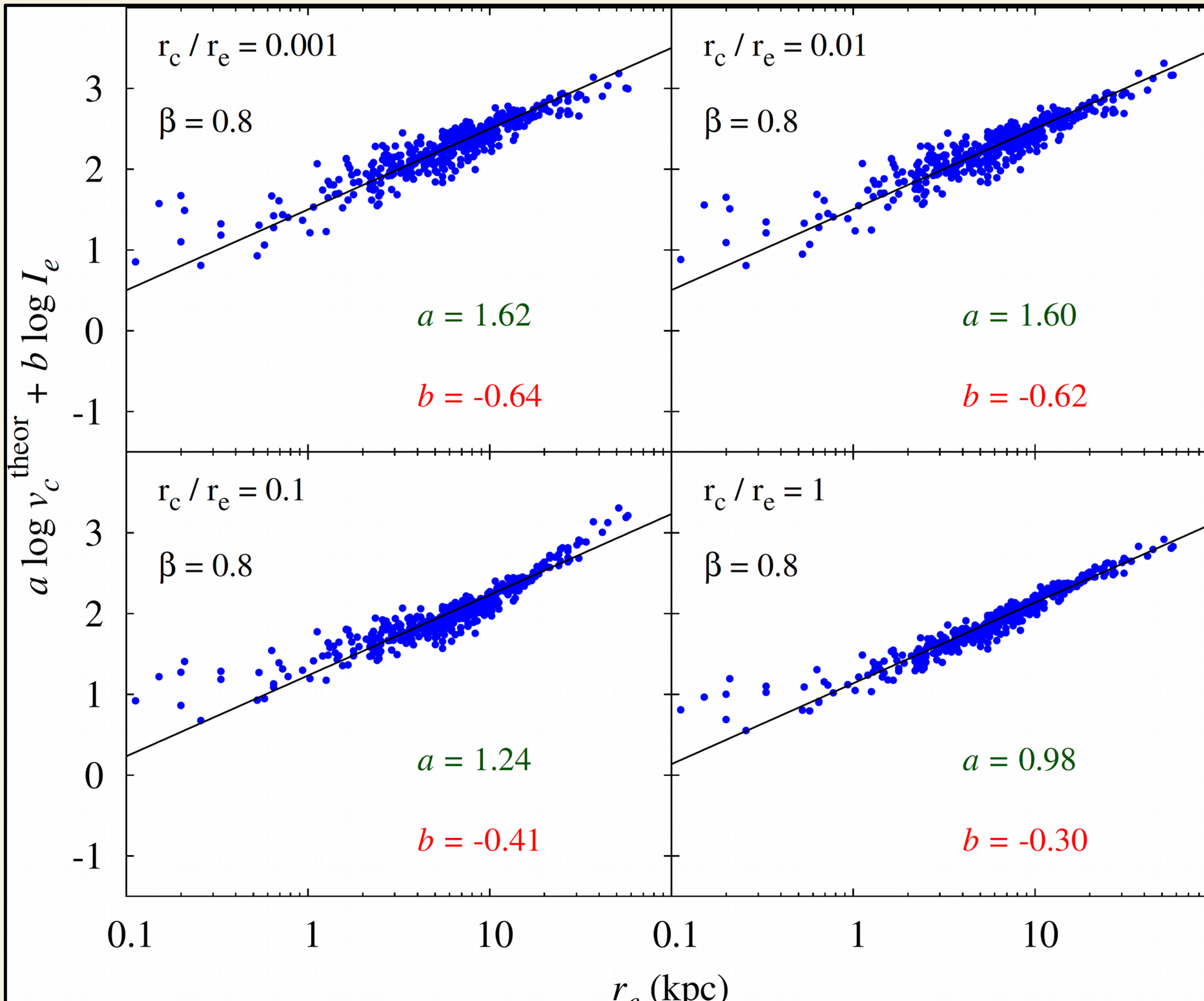


Comparison between the fitted and measured radial velocities for the S2 star.

Some new results

Fundamental plane is empirical relation: $\log r_e = a \log \sigma_0 + b \log I_e + c$

$\boldsymbol{r_e}$ - effective (half-light) radius
$\boldsymbol{\sigma_0}$ - central velocity dispersion
$\boldsymbol{I_e}$ - mean surface brightness within r_e



FP of elliptical galaxies with calculated circular velocity: dependence of FP parameters (a,b) on parameters of R^n gravity.

Conclusions

- For now, General Relativity (GR) is the best theory of gravitation with the largest number of experimentally confirmed predictions, but there is a need to review and extend Einstein Relativity to scales where it has not been properly tested, with aim to better explain observed phenomena at galactic scales, such as flat rotation curves of spiral galaxies, their BTFR, and FP of ellipticals

- *f*(R) theories of modified gravity represent good alternative to DM, and they are good basis to construct an effective theory of gravity
- *f*(R) theories, in particular R^n, can explain (without DM hypothesis) moving of S2 star around SMBH at GC, although the different orbital precession in regard to GR is obtained

- Our future work: revising and extending the theory of gravitational interaction in order to overcome the shortcomings of GR at galactic and cosmological scales (thus, to explain observations and phenomenology without introducing "ad hoc" ingredients).

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Хвала на пажњи!