Network Methods for Big Data: Applications to Astroinformatics

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- Show recent approach of Signal Processing on Graphs/Networks

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- Spectral Graph Wavelets = **Multiscale** Concepts
- Importance of **Multiscale** approach in analysis of Astronomical Images

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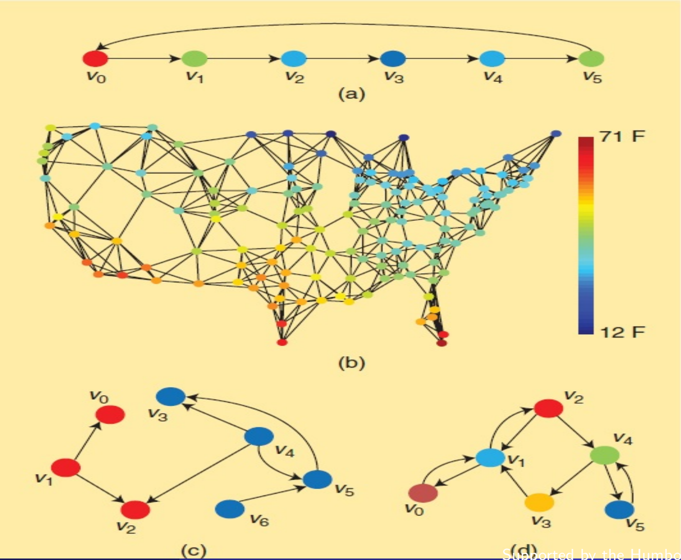
- Challenges: Large-scale **filtering** and **frequency analysis**
- The **new notions** which generalize those of the classical DSP: **Graph signals**, Graph **filters**, Graph **Fourier transform**, Graph **frequency**, Spectrum **ordering**

3 examples of Signals on graphs

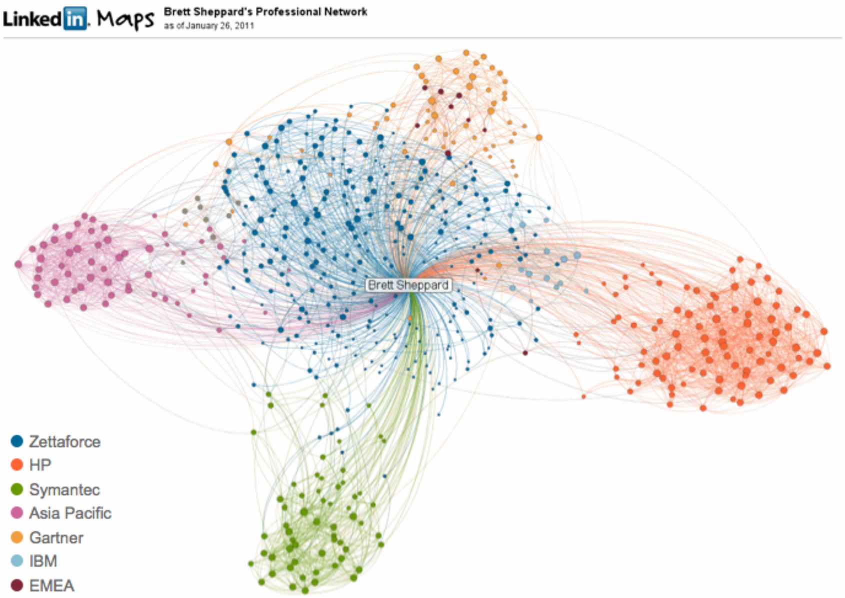
Top: Samples of the signal $\cos(2\pi n/6)$: edges show causality of time;



Examples above:

- **Sensor network** – edges show closeness of locations of the sensors;
- **WWW**: nodes = websites with website features, edges = hyperlinks
- **Social Network**: nodes = individuals, edges = connection and strength
- **Astronomical Images** and their Network approximations
- **Network Approximations** have **nodes** which are galaxies, stars, clusters; nodes are intersection of lines along **edges**; closer Nodes represent **similar features** within the data
- **An Examplary OBJECTIVE of OUR WORK**: **Find a method for high speed automatic classification of galaxies**
- **previously done by thousands of volunteers**
- Application to: **images of distant clusters of galaxies that contain several different types of galaxies**
- General approach of **Machine Learning**: This is the art of "teaching" the algorithms the experience of people accumulated for a long time.
- Applications of the **same methods** e.g. in **medicine**, for helping to spot tumours, or in **security**, to find suspicious items in airport scans.

Example of Network - LinkedIn



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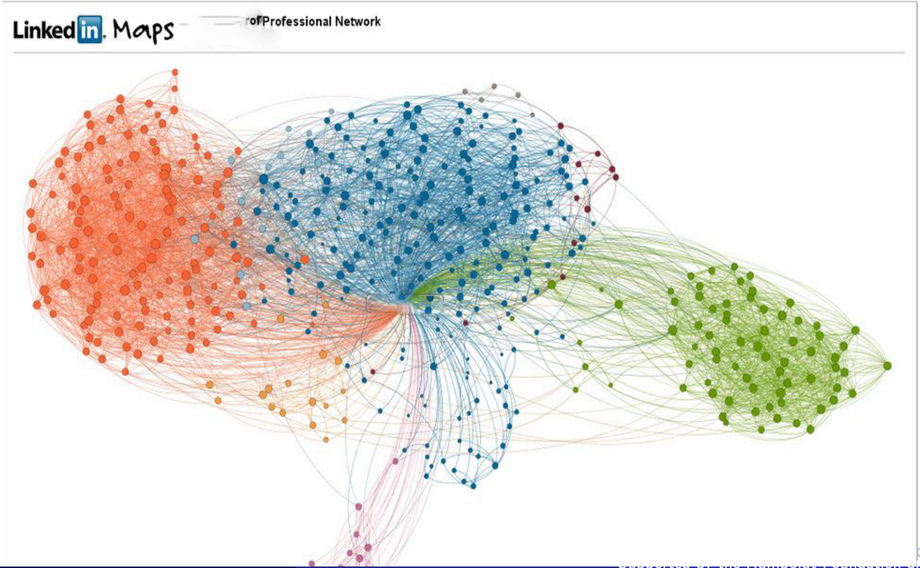
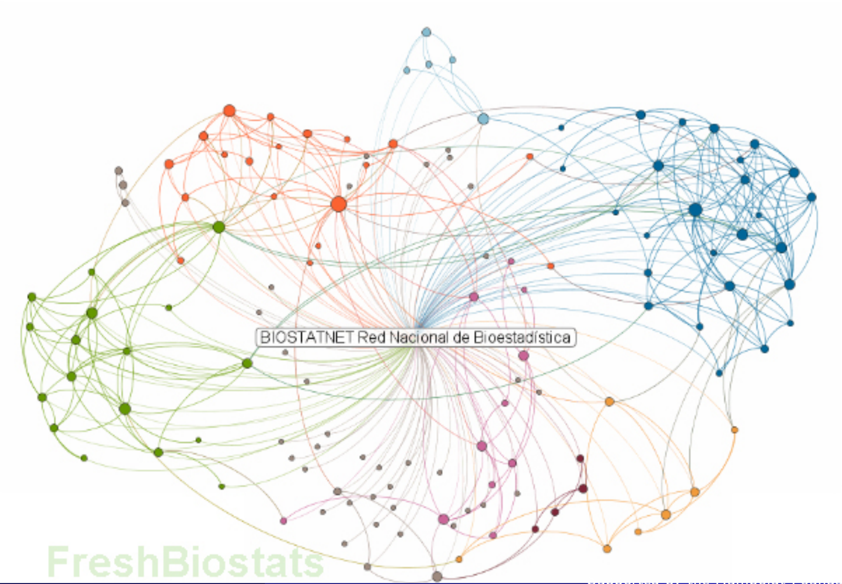
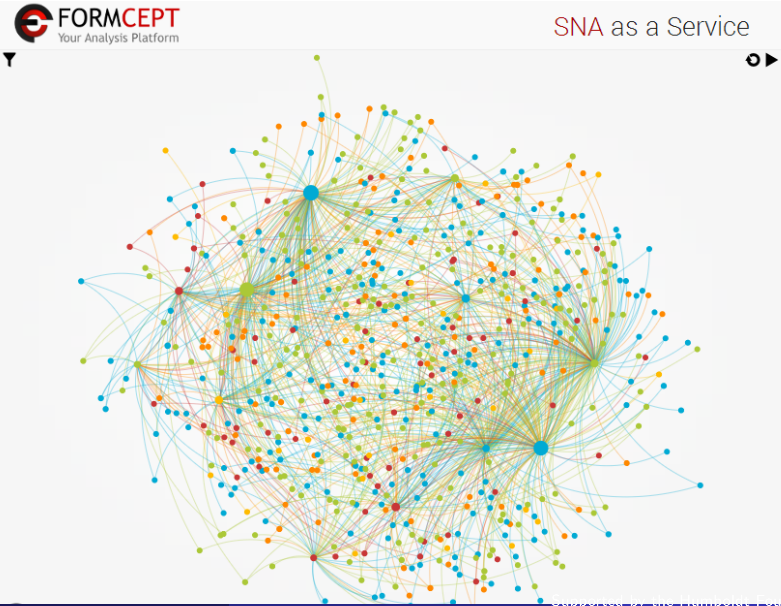


Biostatistics - example graph



Influencer Network-graph



General principles of SP on graphs

- DSP is applied to data sets in which the elements are related by **dependency**, **similarity**, **physical porximity**, etc.

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- The "neighbouring" nodes for the node ν_j is the set

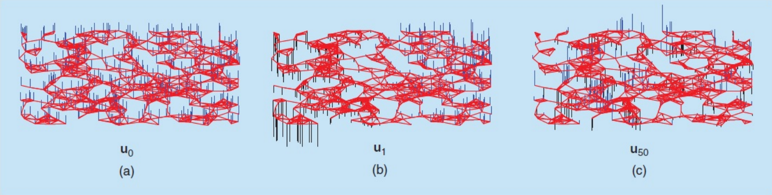
$$N_j$$

Multiscale structure Analysis of Network Signals

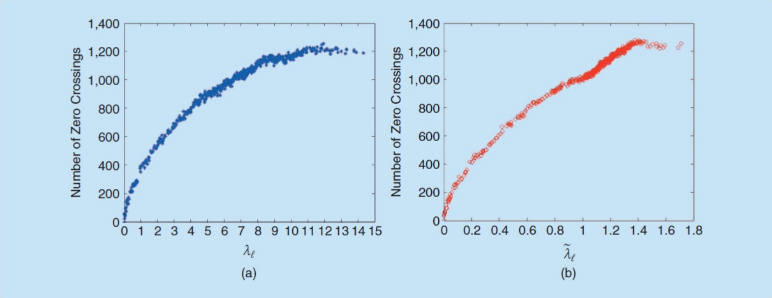
1. Graph shift - generalizes the usual DSP **time delay**; taking average over the Neighbours
$$\overline{s}_n = \sum_{m \in N_n} A_{n,m} s_m$$
2. Graph Fourier transform needs the **Graph Laplace operator** (we assume undirected graph): for a signal f on the graph we define
$$Lf_m := \sum_{n \in N_m} a_{m,n} \left(f_m - f_n \right)$$
3. L is a real symmetric matrix: we denote the N (orthonormal) **eigenvectors and eigenvalues** of L by
$$\chi^{(n)} = \left(\chi_1^{(n)}, ..., \chi_N^{(n)} \right) \in \mathbb{R}^N,$$
$$\lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{N-1}.$$

They are the analogue to e^{ikt}, since $-\frac{d^2}{dt^2} e^{ikt} = k^2 e^{ikt}$

Example of eigenvectors



[FIG2] (a)–(c) Three graph Laplacian eigenvectors of a random sensor network graph. The signals' component values are represented by the blue (positive) and black (negative) bars coming out of the vertices. Note that $\mathbf{u}_{50}$ contains many more zero crossings than the constant eigenvector $\mathbf{u}_0$ and the smooth Fiedler vector $\mathbf{u}_1$.



[FIG3] The number of zero crossings, $|\mathcal{Z}_{\mathcal{G}}(\mathbf{u}_\ell)|$ in (a) and $|\mathcal{Z}_{\mathcal{G}}(\tilde{\mathbf{u}}_\ell)|$ in (b), of the unnormalized and normalized graph Laplacian eigenvectors for the random sensor network graph of Figure 2, respectively (the latter of which is defined in the section "Other Graph

- Hence, we have Fourier Analysis

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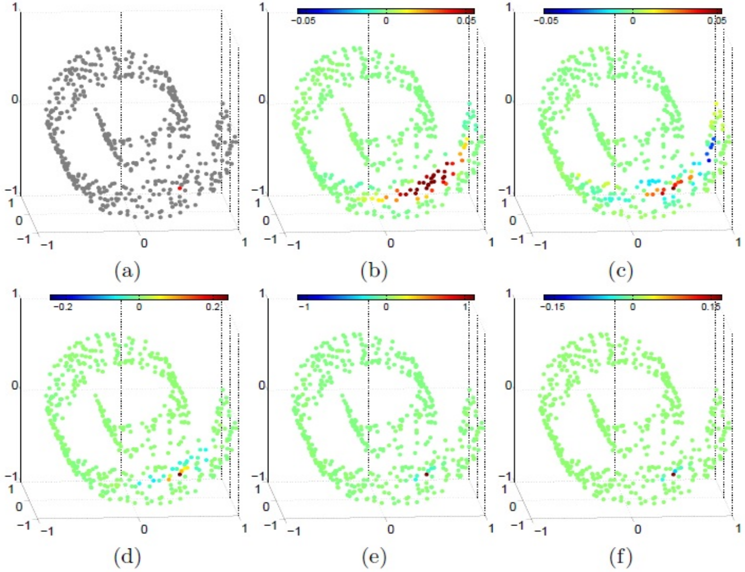
- The role of t - the **scale** of the localized at **node** a wavelet $\psi_{t,a}$ where $\left(a \in G\right)$.

Examples of localizations – Swiss roll



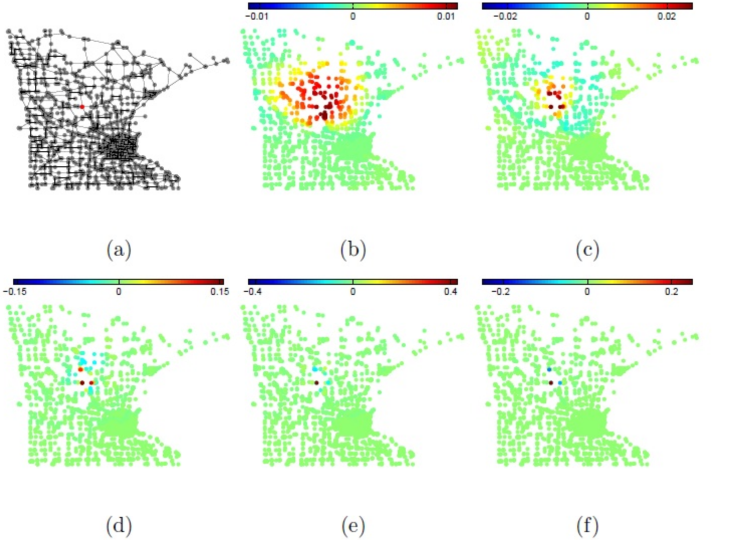


Figure 4: Spectral graph wavelets on Minnesota road graph, with $K = 100$, $J = 4$ scales. (a) vertex at which wavelets are centered (b) scaling function (c)-(f) wavelets, scales 1-4.

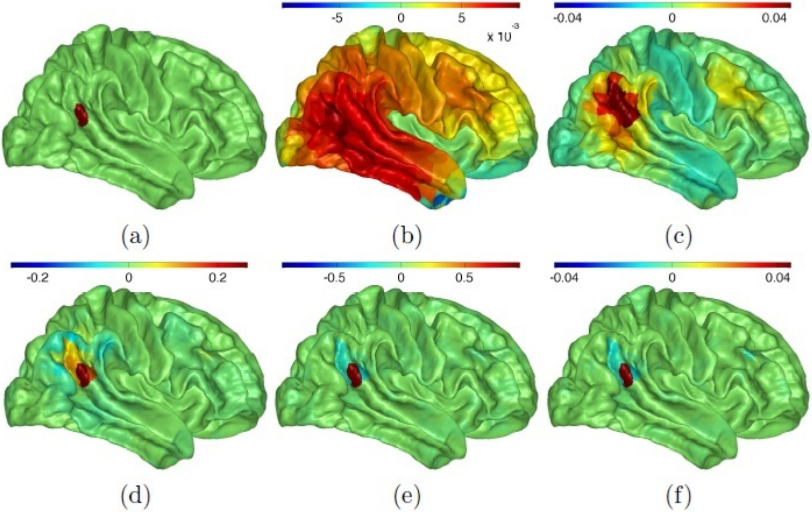


Figure 5: Spectral graph wavelets on cerebral cortex, with $K = 50$, $J = 4$ scales. (a) ROI at which wavelets are centered (b) scaling function (c)-(f) wavelets, scales 1-4.

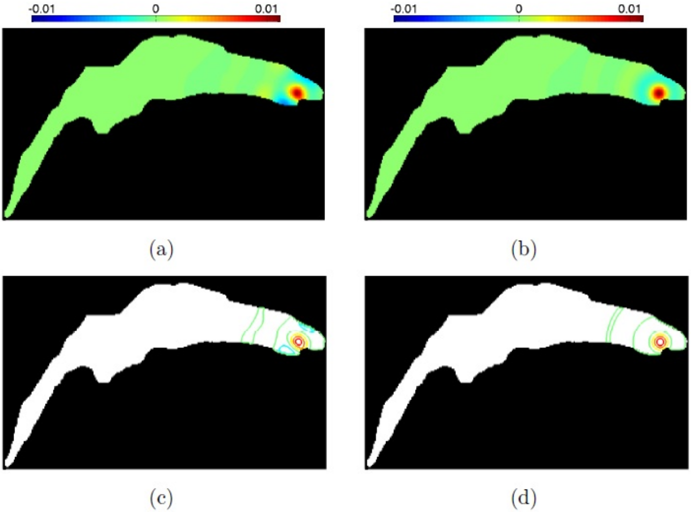
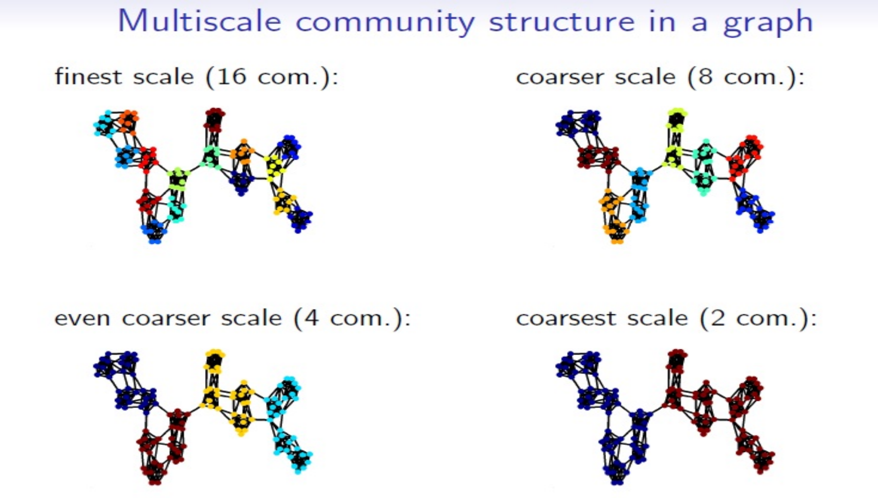
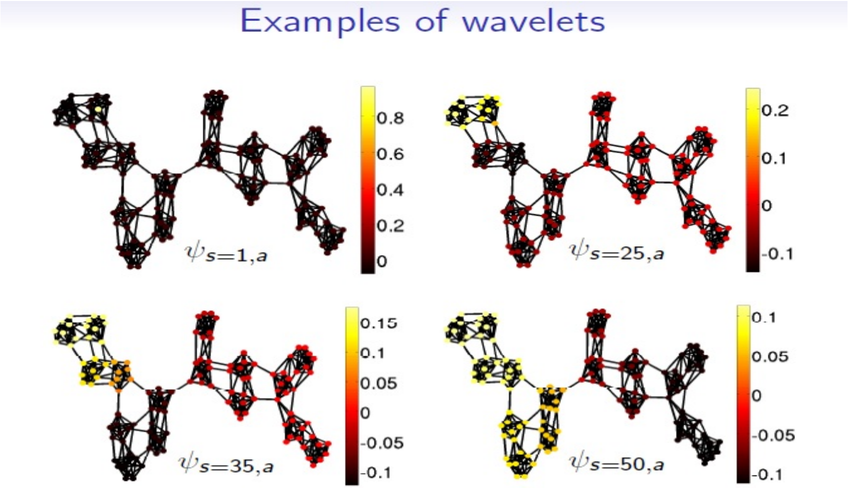


Figure 6: Spectral graph wavelets on lake Geneva domain, (spatial map (a), contour plot (c)); compared with truncated wavelets from graph corresponding to complete mesh (spatial map (b), contour plot (d)). Note that the graph wavelets adapt to the geometry of the domain.

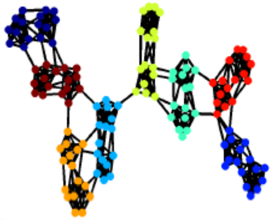
Main Objective: Clusterization (Community Detection on graphs)




Examples of wavelets
$\psi_{s=1,a}$
0.8
0.6
0.4
0.2
0
$\psi_{s=25,a}$
0.2
0.1
0
-0.1
$\psi_{s=35,a}$
0.15
0.1
0.05
0
-0.05
-0.1
$\psi_{s=50,a}$
0.1
0.05
0
-0.05
-0.1

Comparison with other methods for Community Detection

Classical community detection algorithm do not have this "scale-vision" of a graph. Modularity optimisation finds:



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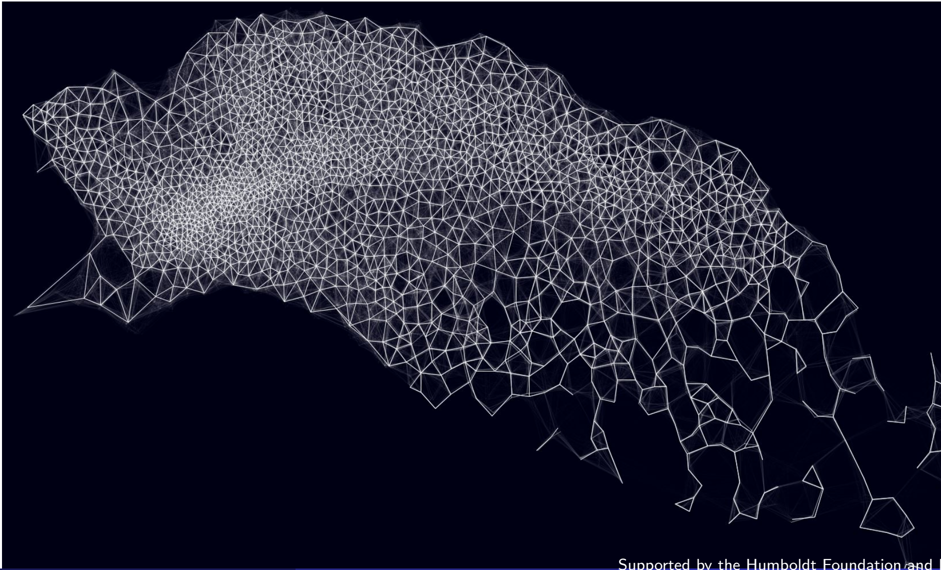
- Astronomical image segmentation by Networks and wavelets
- Machine learning method to analyse galaxy images

Pictures to:

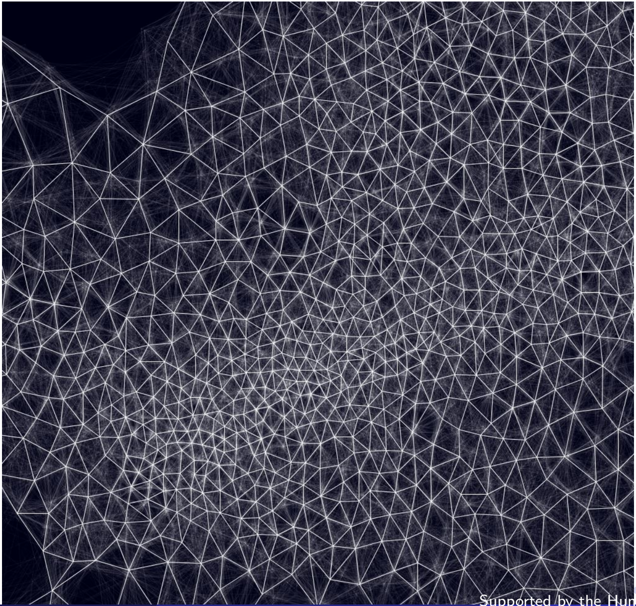
Machine learning: to "teach" a machine to analyse galaxy images

1. Picture 1: Hubble Space Telescope image of the **cluster** of galaxies MACS0416.1-2403, one of the Hubble 'Frontier Fields'. Bright yellow 'elliptical' galaxies can be seen, surrounded by numerous blue **spiral** and amorphous (star-forming) galaxies
2. Picture 2: Visualisation of the **(neural) network** representing the 'brain' of the machine learning algorithm. The intersections of lines are called nodes, and these represent a map of the input data. Nodes that are closer to each other represent similarity
3. Picture 3: A **zoom-in of part** of the network described above. Credit: J. Geach / A. Hocking
4. Picture 4: Image showing the MACS0416.1-2403 **cluster**, highlighting parts of the image that the algorithm has identified as '**star-forming**' galaxies. Credit: NASA / ESA / J. Geach / A. Hocking
5. Picture 5: Image showing the MACS0416.1-2403 **cluster**, highlighting parts of the image that the algorithm has identified as '**elliptical**' galaxies. Credit: NASA / ESA / J. Geach / A. Hocking

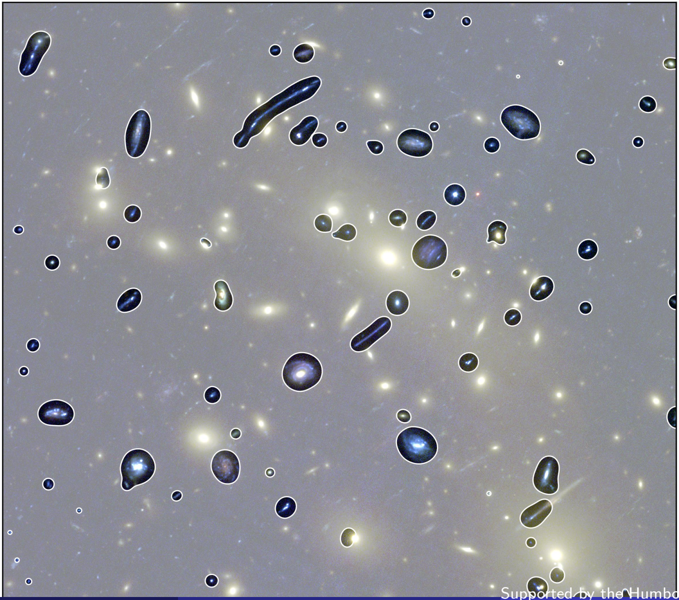
Continued - Network approximation



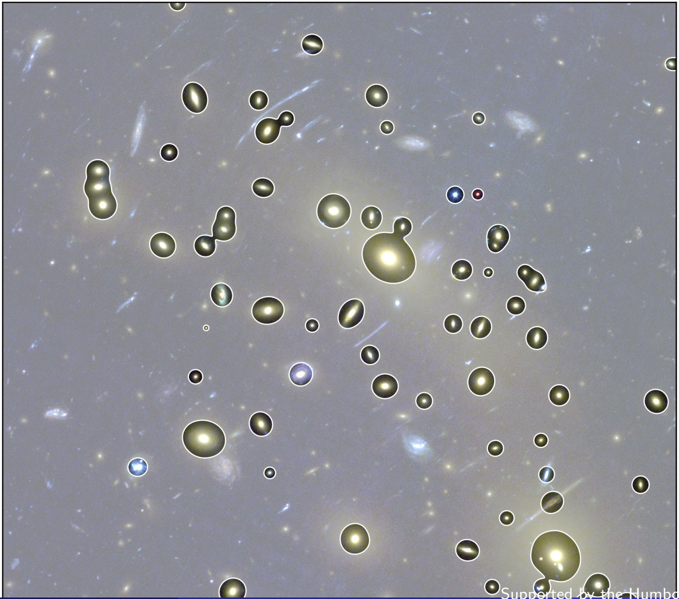
Cont'd - zoomed part



Cont'd - "star forming" galaxies



Cont'd – "elliptical" galaxies



Deep Learning and Wavelets approach

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