

*The Broadening of Spectral Lines  
by Collisions with Neutral  
Hydrogen Atoms in Cool Stars*

**Paul Barklem**

**Department of Astronomy and Space Physics  
Uppsala University**



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# Acknowledgements

- Jim O'Mara
- Stuart Anstee
- Jenny Aspelund-Johansson
- Boutheina Kerkeni
- Nicole Feautrier
- Annie Spielfeldel

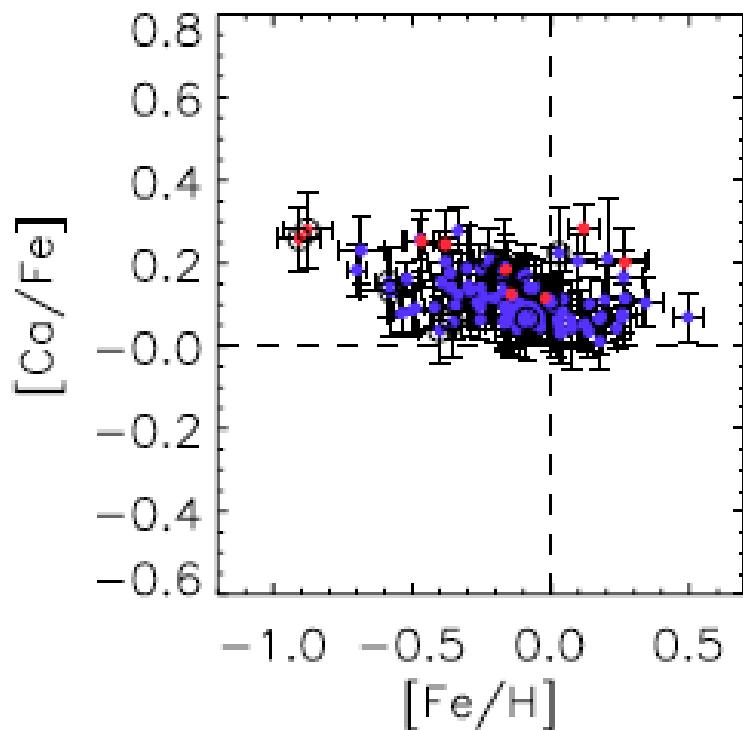
# Scientific Motivation

Need accurate stellar chemical abundances from spectroscopy for many problems in modern astrophysics:

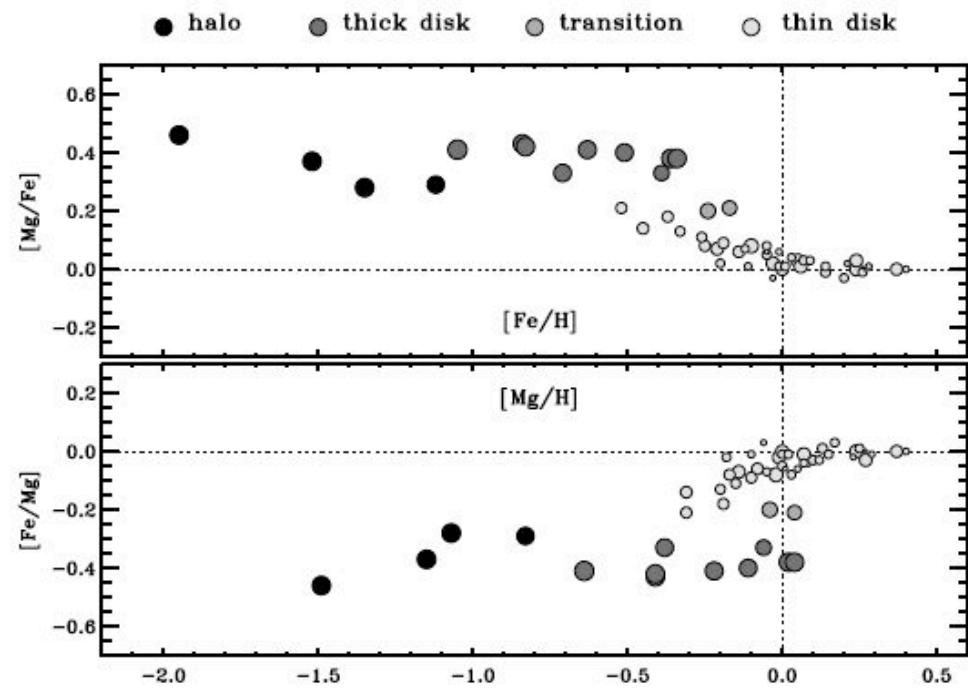
- Understanding the chemical and dynamical evolution of the Galaxy, e.g.:
  - Origin and evolution of bulge, thin and thick disks
  - Merger history
  - Astrophysical sites of nucleosynthesis processes
- Solar composition and its place in the solar neighbourhood
- Strong lines are often the best to use - in some cases, e.g. very cool stars, galaxies, there is no choice

# Scientific Motivation

Solar neighbourhood



Galactic Thick and Thin Disks



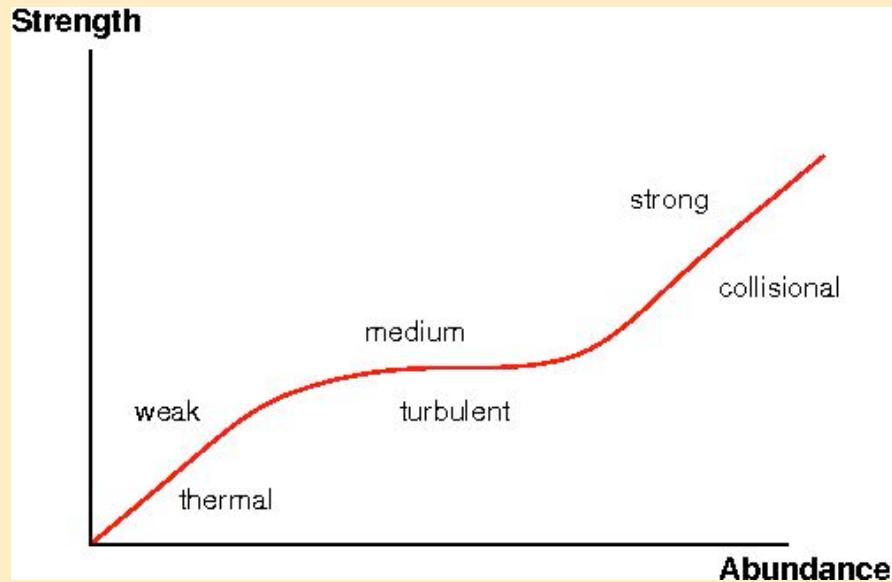
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# Scientific Motivation



Weak lines are susceptible to blends

Medium strength lines are saturated and insensitive to abundance

Strong lines typically have the best oscillator strengths, and often no choice:  
E.g. very cool stars, galaxies, distant stars

Cool star atmospheres dominated by neutral hydrogen, in its ground state

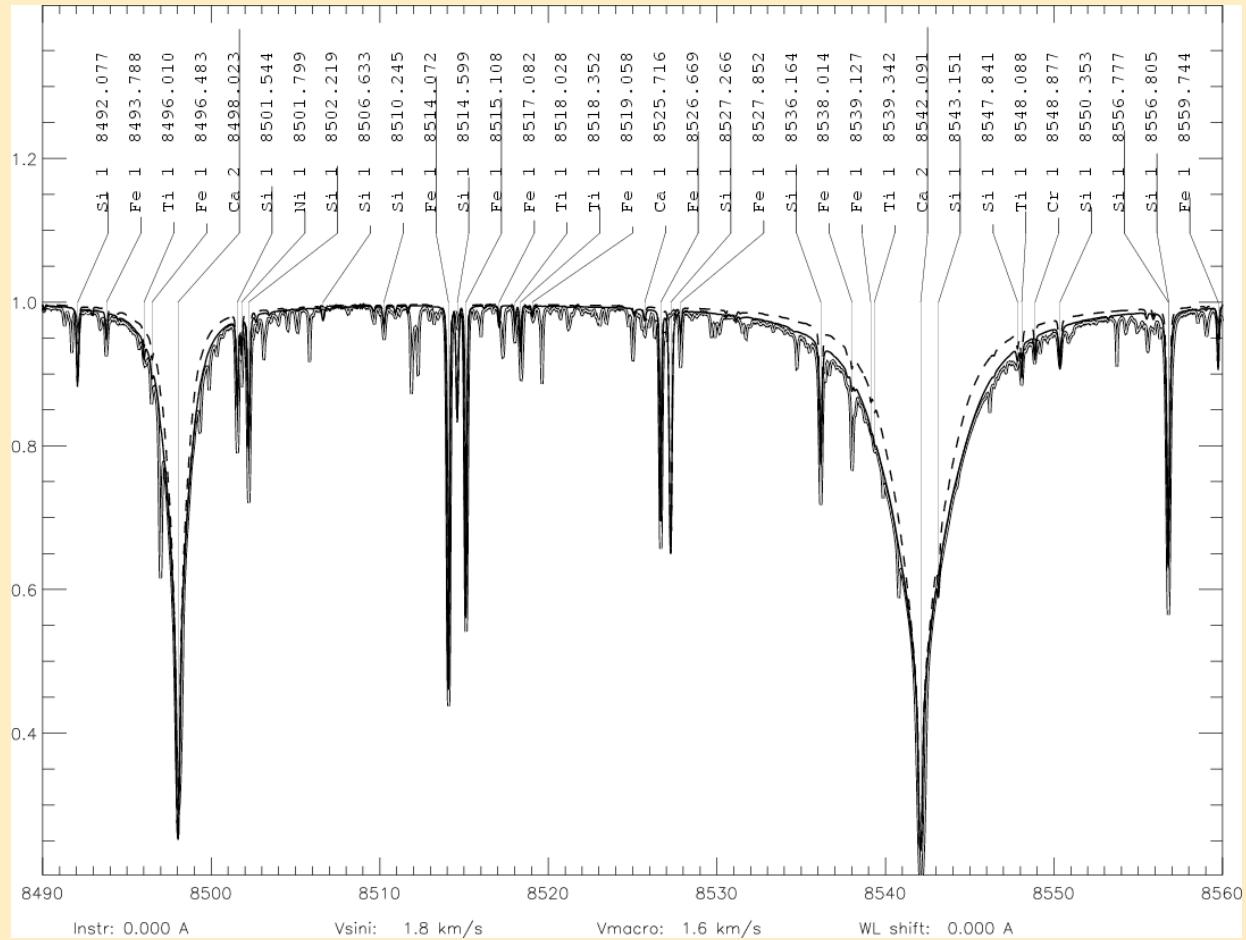
90% H, 9% He, 1% metals

$$\frac{N_H}{N_e} \approx 10^4$$

Weight of number means generally dominant over electrons

# Scientific Motivation

- E.g. Ca II IR triplet in the Sun



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# History

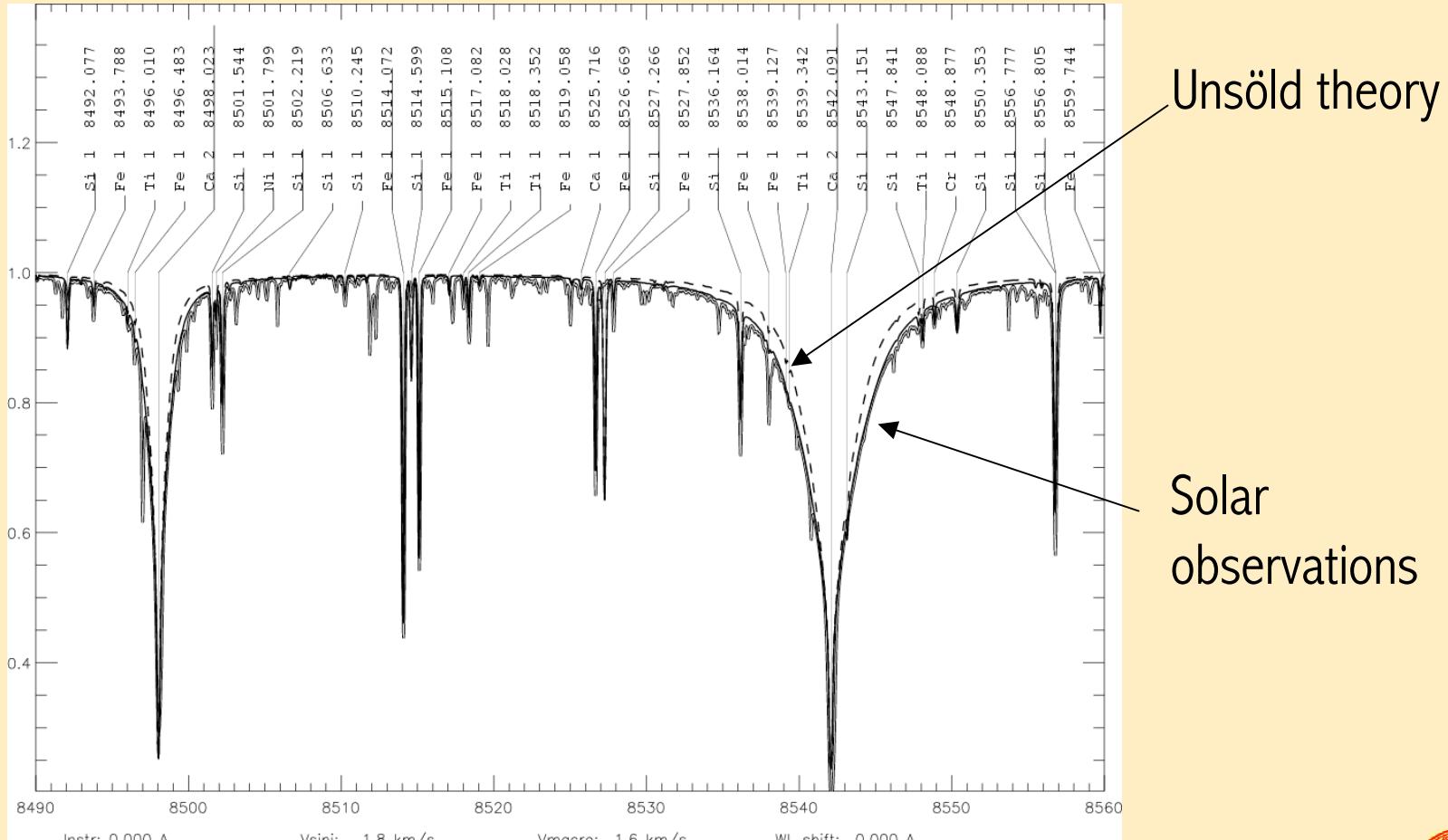
- Lorentz (1906) Impulse theory (prior to QM)
- Weisskopf/London (1930's) VdW potential, strong collisions
- Lindholm/Foley (1940's) weak collisions, averaged interaction
- Unsöld (1950's) apply VdW + approx, general formula

$$\text{VdW} = \text{van der Waals} \quad \Delta E \approx C_6 / R^6$$

$$\Gamma = 17v^{3/5}C_6^{2/5}N_H$$

- (1960's -) Accurate calculations for a few specific cases

# Scientific Motivation



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# Scientific Motivation

- Astrophysical evidence that Unsöld theory is inadequate
- Astrophysics needs a theory to compute data for a large number of lines of various elements (e.g. Fe, Ni, Mg, Ca, etc)
- Theory should ideally be simple to use!

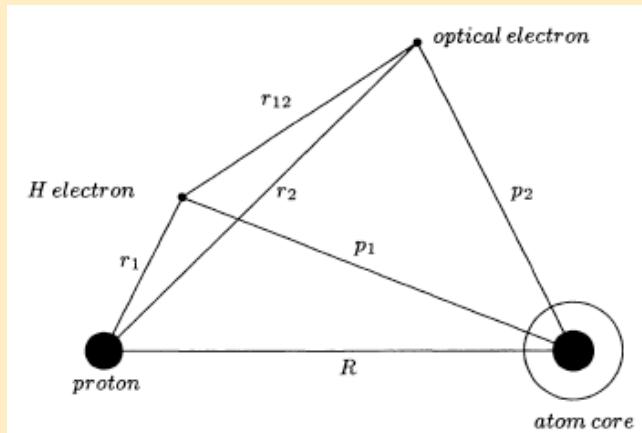
# History

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- Unsöld (1950's) apply VdW + approx, general formula
  
- Brueckner (1970's) numerical RSU perturbation theory
- O'Mara (1970's) analytic RSU perturbation theory
- Anstee & O'Mara (1992) removal of averaged interaction+more
- Barklem & O'Mara (1998-) extension to d and f states, and ions

# ABO theory

- isolated lines
- impact approximation (Lorentzian profile)
- classical straight path approximation
- no fine structure (orbital angular momentum basis,  $n/lm$ )
- no quenching
- Important part is the potentials:

# ABO theory: RSU potentials



$$V = \frac{1}{R} + \frac{1}{r_{12}} - \frac{1}{r_2} - \frac{1}{p_1}$$

Covalent partitioning  
(Ionic partitioning neglected)

$$\Delta E = \langle i | V | i \rangle + \sum_{j \neq i} \frac{\langle i | V | j \rangle \langle j | V | i \rangle}{E_i - E_j}$$

Rayleigh-Schrödinger Perturbation Theory  
to 2nd order (no exchange)

$$\approx \langle i | V | i \rangle + \frac{1}{E_p(R)} \langle i | V^2 | i \rangle$$



Unsöld approximations

$$\approx \langle i | V | i \rangle + \frac{1}{E_p} \int_0^{\infty} R_{nl}^2(p_2) I_{lm}(p_2, R) p_2^2 dp_2$$

$$E_p(R) \approx E_p(\infty)$$



Analytic functions where all integrals except the final integral  
over the radial wavefunction have been performed

# ABO theory: RSU potentials

$$\Delta E = \langle i | V | i \rangle + \sum_{j \neq i} \frac{\langle i | V | j \rangle \langle j | V | i \rangle}{E_i - E_j}$$

$$\approx \langle i | V | i \rangle + \frac{1}{E_p(R)} \langle i | V^2 | i \rangle$$

$$\approx \langle i | V | i \rangle + \frac{1}{E_p} \int_0^\infty R_{nl}^2(p_2) I_{lm}(p_2, R) p_2^2 dp_2$$

- Coulomb wavefunctions ( $n^*$ ,  $l$ )
- Unsöld approx  $E_p = -2/\alpha_H = -4/9$
- Cross sections depend only on  $n^*, l \rightarrow$  Independent of species!

$$n^* = [2E_{binding}]^{-1/2}$$



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# ABO theory: Results

Cross section

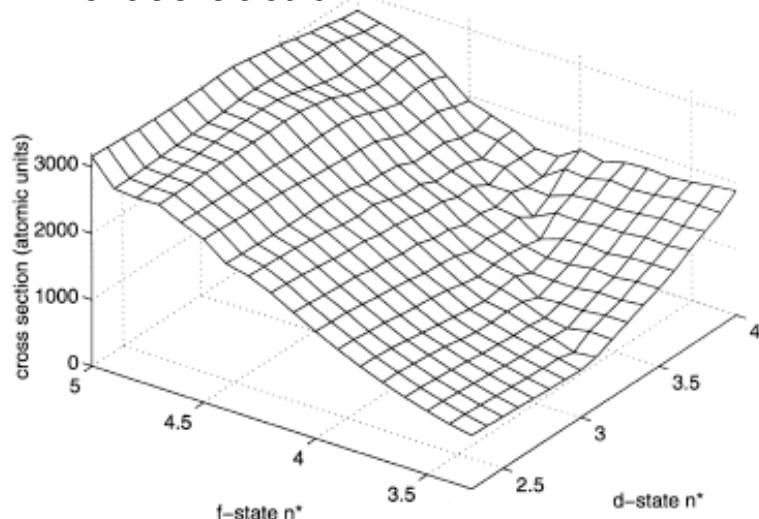


Figure 1. Plot of the cross-section for a perturber velocity of  $10^4 \text{ m s}^{-1}$  against the effective principal quantum number of the two states.

Velocity dependence

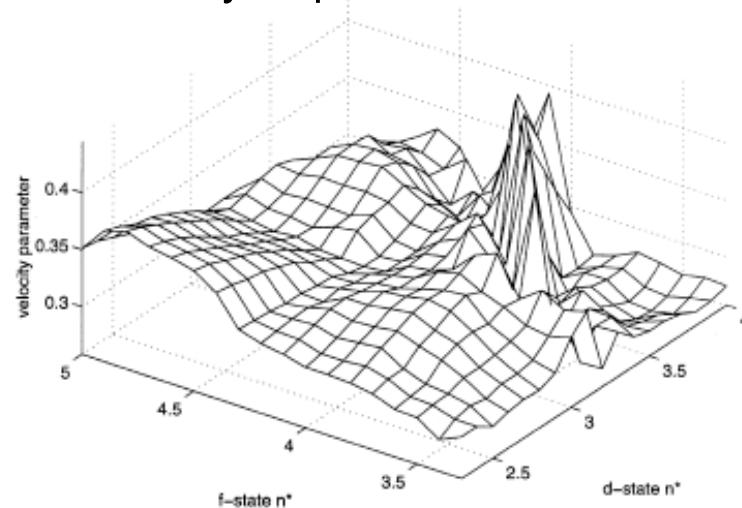


Figure 2. Plot of the velocity parameter  $\alpha$  against the effective principal quantum number of the two states.

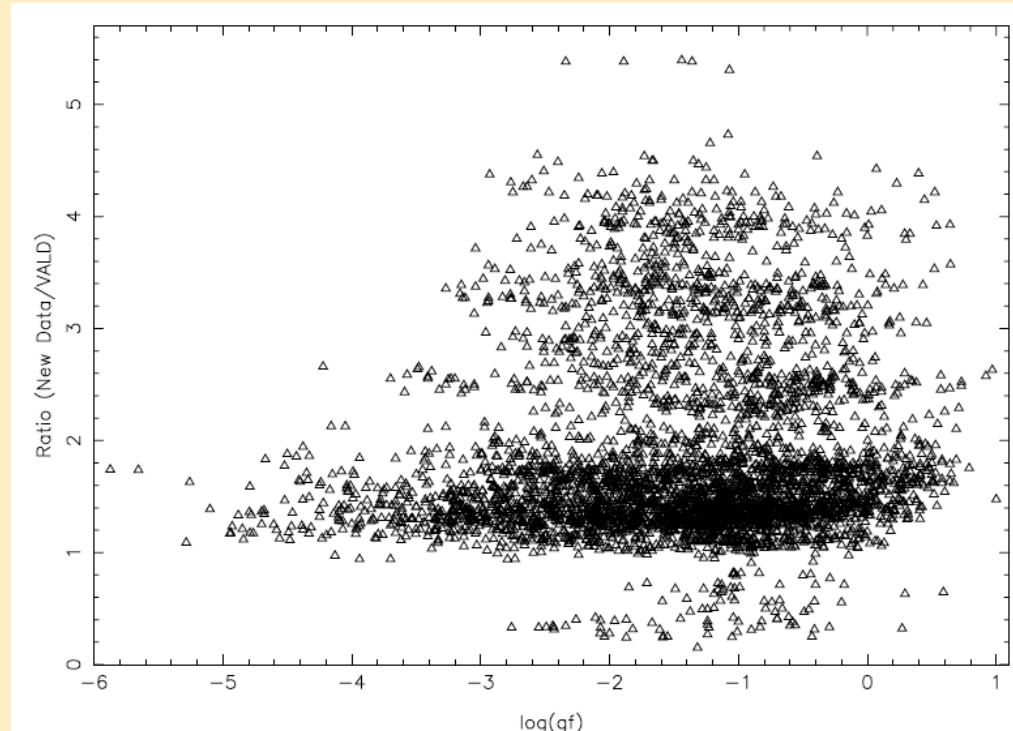
Tabulated results dependent only on effective principal quantum number

Simple velocity behaviour:

$$\sigma(v) = \sigma(v_0) \left( \frac{v}{v_0} \right)^{-\alpha}$$

# ABO theory: Results

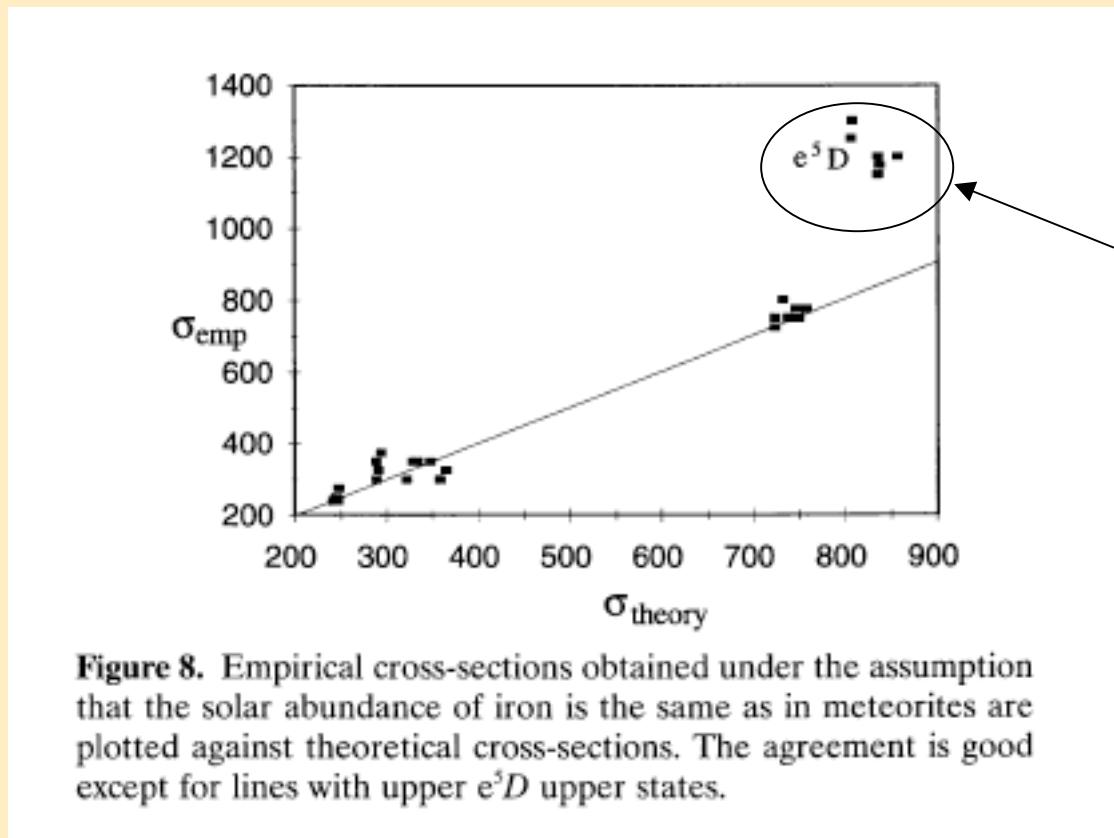
Comparison with Unsöld's theory for 4891 lines, Li to Ni:



Mean difference of 1.88, in rough agreement with astrophysical fudge factor commonly used of 2

# Applications

- Application to lines of Na, Ca and Fe in the solar spectrum indicate uncertainties as low as 5%



**Figure 8.** Empirical cross-sections obtained under the assumption that the solar abundance of iron is the same as in meteorites are plotted against theoretical cross-sections. The agreement is good except for lines with upper  $e^3D$  upper states.

# Testing

- Comparison with more detailed calculations indicates uncertainties of order 5-20%.

**Table 1.** Comparison of line widths per unit perturber density ( $w/n_H$  in units  $10^{-8} \text{ cm}^3 \text{ rad s}^{-1}$ ) at 5000 K for resonance lines of Mg, Ca and Sr using different potentials and dynamics.

Dynamics →	Quantal	Semi-classical	Semi-classical	Semi-classical
Potentials →	MOLPRO	MOLPRO	ABO	Hybrid
Mg	1.13	1.10	1.01	1.25
Ca	1.23	1.24	1.10	1.28
Sr	1.49	1.48	1.18	1.48

No difference due to dynamics

10-20% due to potentials  
-neglect of ionic crossing

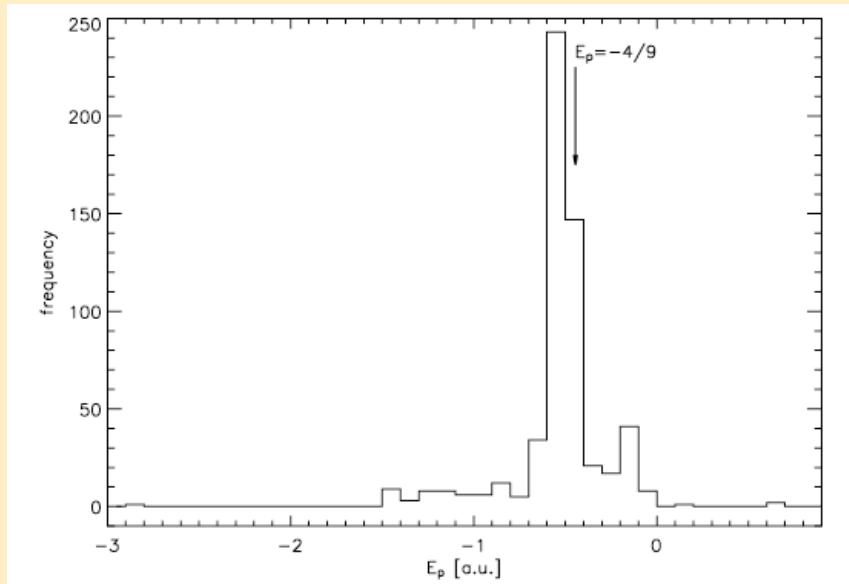
# Extension to ions

- Unsöld approx  $E_p = -4/9$  is no longer valid
- Compute via

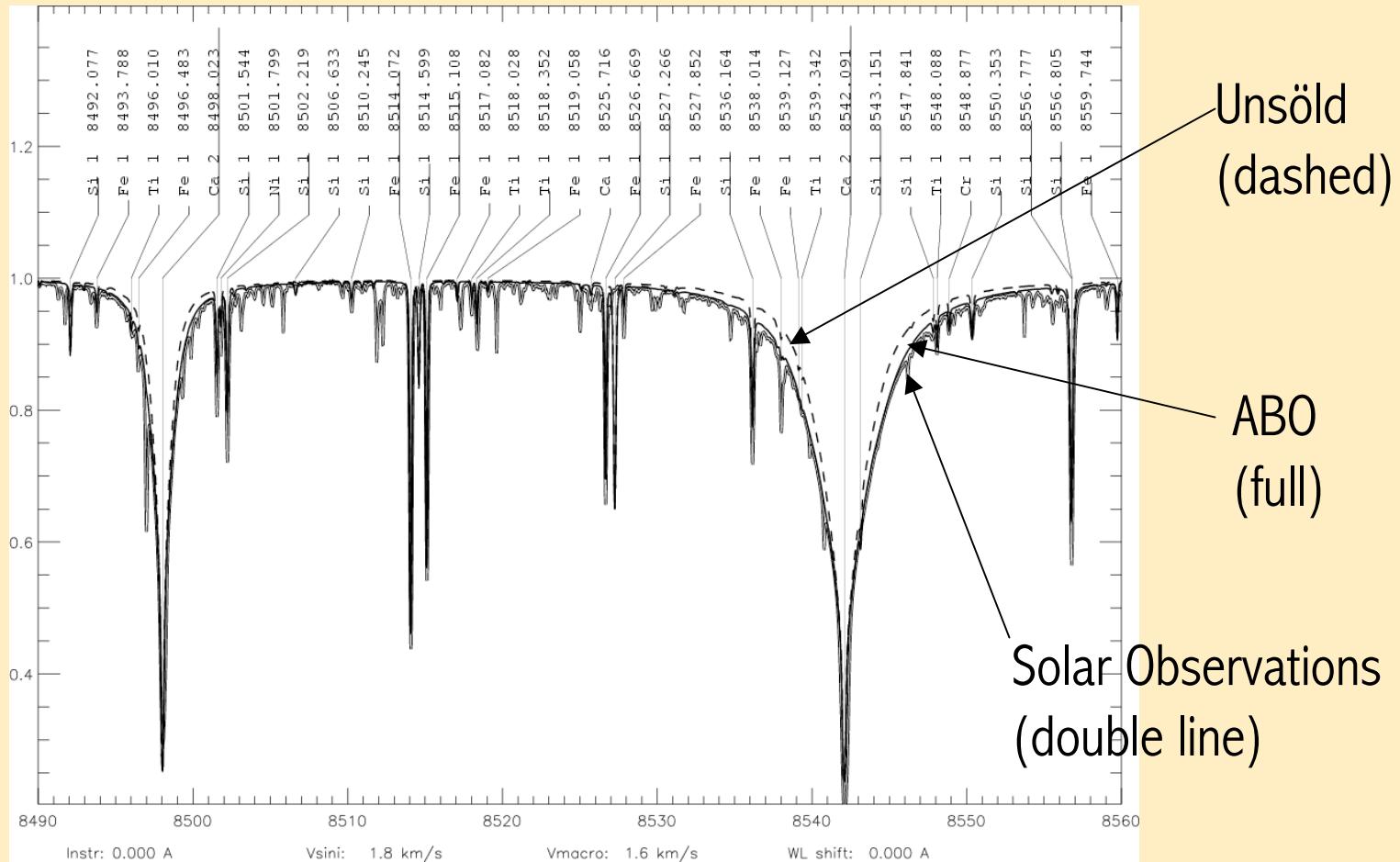
$$C_6 = \frac{3}{2} \sum_{k' \neq k} \sum_{l' \neq l} \frac{f_{kk'}^A f_{ll'}^H}{(\Delta E_{k'k}^A + \Delta E_{l'l}^H) \Delta E_{k'k}^A \Delta E_{l'l}^H},$$

$$E_p = -\frac{2 \langle p_2^2 \rangle}{C_6}.$$

Large Scale calculations for Fe II  
using large scale semi-empirical  
atomic data calculations by Kurucz



# Applications



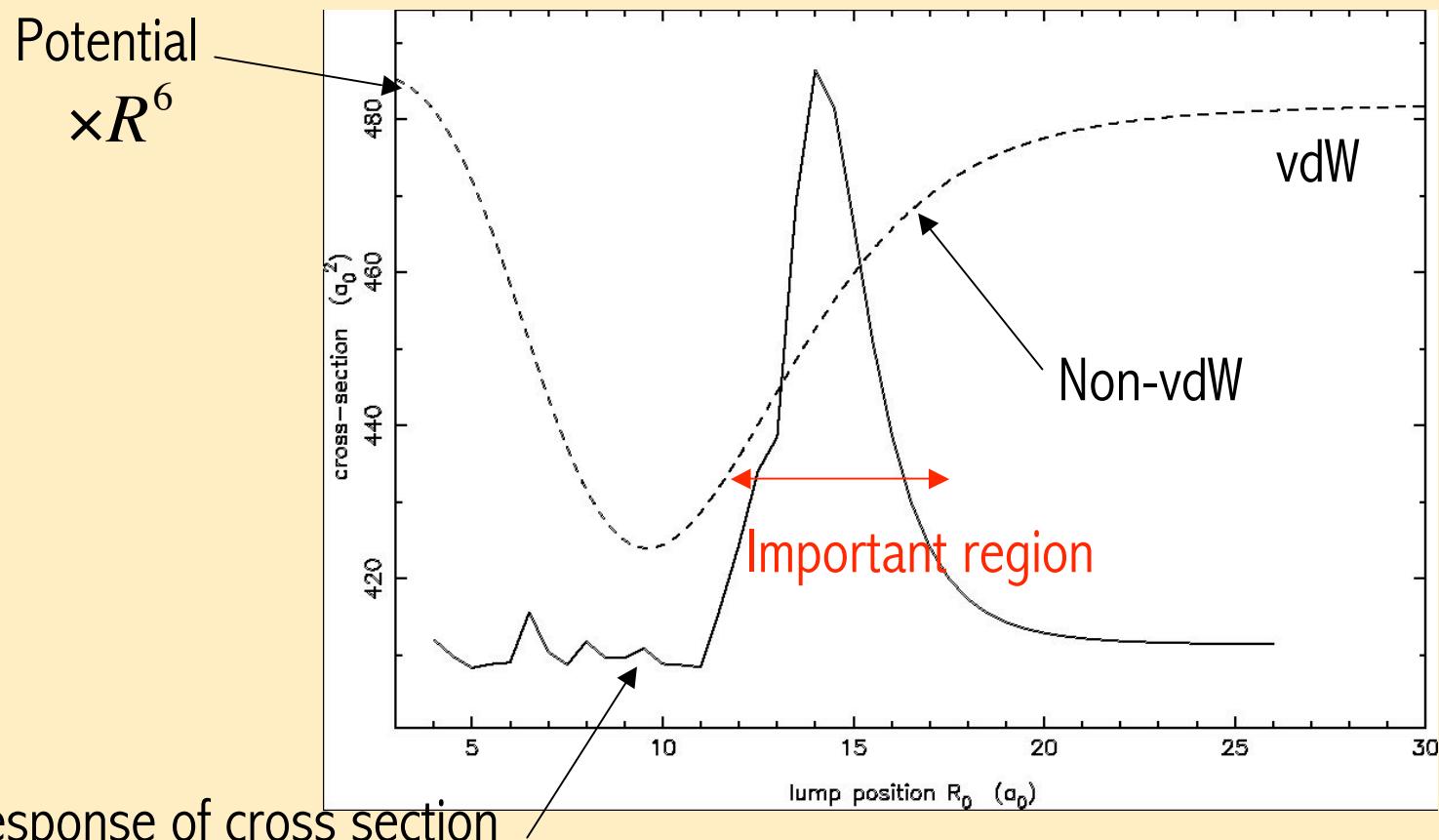
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# ABO theory: Why it works

“Intermediate” separations most important:



Response of cross section  
To local perturbation in potential

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# ABO theory: Why it works

Often, avoided ionic crossings either diabatic, or in the strong collision regime:

Crossing or  
Weisskopf  
Radius

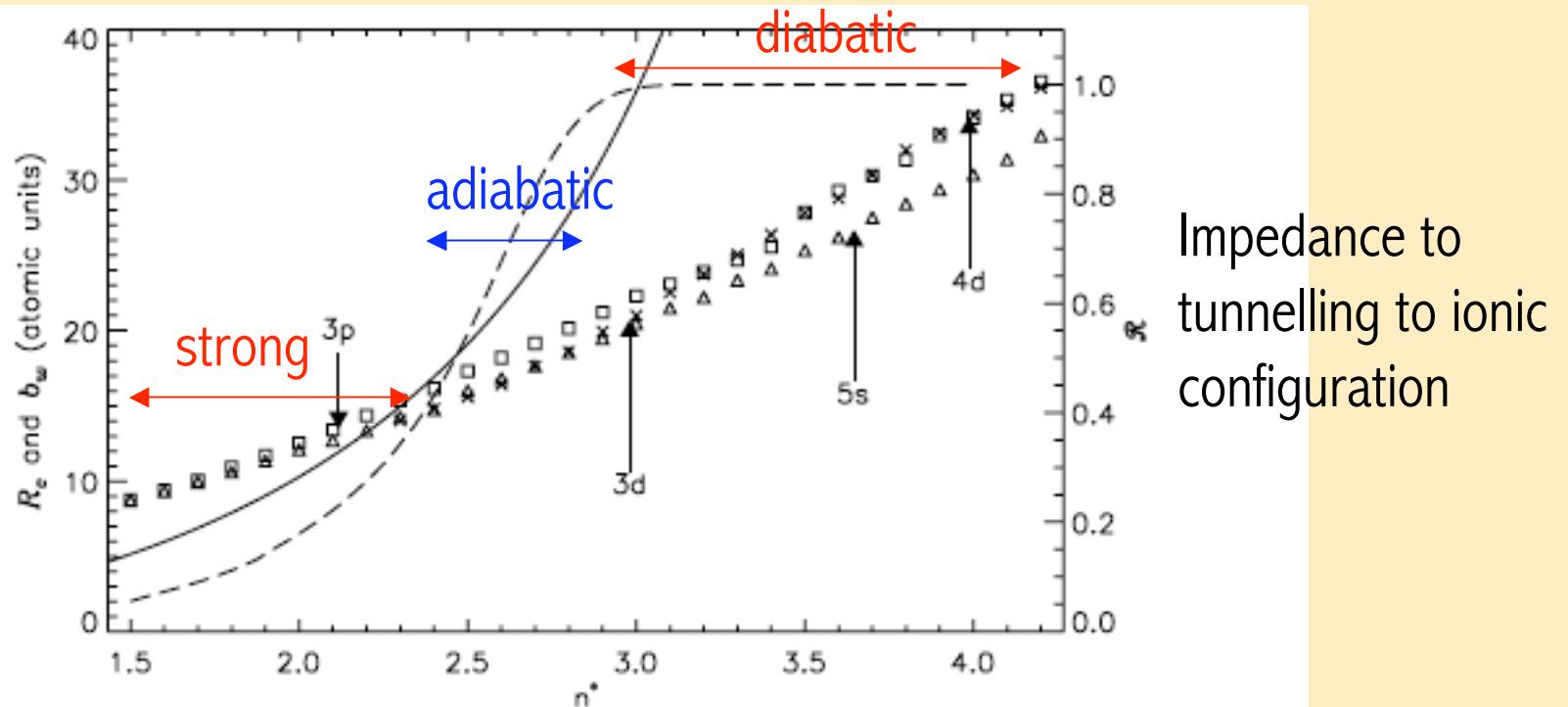


Figure 4. Plots demonstrating various collision regimes.  $b_w$  plotted against  $n^*$  for s states (triangles), p states (squares) and d states (crosses) for  $v = 10^4 \text{ ms}^{-1}$ . The full curve is  $R_e$ . The reflection coefficient  $\mathcal{R}$ , is plotted as the dashed curve.

# Summary of Results

- Tables of general data for transitions in neutrals involving s, p, d and f states
- Code for interpolating in tables available
- Table of 4891 strong lines, Li to Ni, mostly neutral + important ionised lines
- Table of 24188 Fe II lines
- Table of 13167 Cr II lines (unpublished)
- Also extended to H Balmer lines

# Conclusions

- ABO theory provides a general and widely applicable theory, with accuracy of better than 20%
- Strong lines can now be used with confidence in analysis of cool star spectra (no fudge factors)
- Theory is gaining wide use
- Future: inclusion of ionic state should lead to some improvement