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ON THE APPLICATION OF BALMER BETA LINE SHAPE FOR ELECTRON DENSITY DIAGNOSTICS IN THE RANGE $10^{20} - 10^{21}$ m^{-3}

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1. INTRODUCTION

The analysis of hydrogen Balmer beta (H_β = 486.13 nm) spectral line shape is one of the most common methods of electron density diagnostics from the beginning of seventies. Although, the laser interferometry method seems to provide the most reliable results at present, analysis of H_β line shapes is the easiest one and can be performed in every spectroscopic laboratory with satisfactory accuracy. In practice the most frequently used approaches are: the approximate experimental formulas and the comparison with tabulated data according to unified theory - VCS tables (Vidal,Cooper,Smith,1973), which is already carefully experimentally tested. In this work we present critical analysis of applications of this methods in case of relatively low electron concentrations (10^{20} - 10^{21} m^{-3}).

2. EXPERIMENTAL APPROXIMATE FORMULAS

On the basis on electron density (Ne) measurements via laser interferometry and determination of half – half widths (HWHM) under identical experimental conditions few experimental approximate formulas for dependence Ne = f (HWHM) are obtained (Wiesse et al, 1972, Kelleher, 1981). By application of these formulas on HWHM determinated from VCS tables at Ne = $1*10^{20}$ m^{-3} and Ne = $1*10^{21}$ m^{-3} for various temperatures, testing was performed. It should be stressed, that HWHM obtained by unified theory shows the best agreement with experimental data.

2.1 *Wiesse's formula*

From the measurements in electron density range 1.5 * 10^{22} - 10^{23} cm^{-3} and temperature range 9 000 – 14 000 K following approximate formula is determinated

$$Ne\ (cm^{-3}) = 10^{16} * (HWHM / 4.74)^{1.49}$$

i.e. according to (Helbig, 1998)

$$Ne\ (cm^{-3}) = 10^{16} * (FWHM / 9.4659)^{1.48983}$$

where HWHM is pure Stark halfwidth. Although HWHM very slowly depends on electron temperature Te, for Ne <$1*10^{21}$ m^{-3} and Te > 10 000 K errors become greater than 10%. Also, the main problem with application of this formula is complicated procedure of profile deconvolution at lower densities and consequently lower accuracy.

2.2. *Kelleher's formula*

From the measurements in Ne range 0.2 - 1.3 * 10^{22} m^{-3} and T from 10 000 to 20 000 K Kelleher obtained:

$$Ne\ (cm^{-3}) = 10^{16} * (W_S / 4.95)^{1.48}$$

where Ws - Stark halfwidths, W_D - Doppler halfwidths and Wi - instrumental halfwidths

$$W_S = (W_m^{1.4} - W_{D,i}^{1.4})^{1/1.4} \qquad W_{D,i} = (W_D^2 + W_i^2)^{0.5} \qquad W_D = 3.58 * 10^{-7} \lambda (T_g / M)^{0.5}$$

As one can see, the corrections due to instrumental and Doppler broadening in this formula are included. Unfortunately, validity range and origin of this correction is not known. This formula shows lower accuracy than Wiesse's under the same test conditions. Other authors (Czernichowski et all,1985) obtain similar conclusion. Namely, if a value of Ne obtained by Wiesse's formula instead of this one is used, all Kelleher's data (on shape of He I 447.1 nm spectral line from the same paper) are in accordance with papers published by other authors.

3. APPLICATIONS OF VCS TABLES

The VCS tables contain the normalized profiles $S(\Delta\alpha)$, from which one can obtain $S(\Delta\lambda)$ by

$$\Delta\lambda = F_0 \Delta * \alpha, \qquad S(\Delta\lambda) = S(\Delta\alpha) / F_0 \qquad \text{and} \qquad F_0 = 1.25 * 10^{-9} Ne^{2/3}.$$

where $\Delta\lambda$ is the wavelength perturbation of a line with respect to the unperturbed position of the line (in Å) and F_0 the normal field strength (in esu) due to electrons with density Ne (number per cm^{-3}).

3.1. *Graphic method*

From the obtained $S(\Delta\lambda)$ profiles, after determinations of halfwidths, the theoretical dependence of Ne on HWHM in log-log proportion for various temperatures are drawn. From such graphics for experimentally obtained halfwidths Ne can be obtained. This method is the easiest one, but one must have in mind that error of 7 % in determination of Ne requires determination of log(HWHM) with error less than 1%
(for example $\log (1.07*10^{21}) = 21.03$ with error of 0.14 %).

3.2. *Approximate formula Czernishowski – Chapelle*

On the basis of VCS tables the approximate formula (Czernichowski et all,1985) with accuracy of 5 % in the range $0.0316 < Ne[10^{22}\ m^{-3}] < 3.16$ and $5000 < Te[K] < 20000$ are obtained:

$$\log Ne = 22.578 + 1.478 \log W - 0.144 (\log W)^2 - 0.1265\ T$$

where W is FWHM of whole profile in nm, T is excitation temperature in K and Ne in m^{-3}. This formula has a 5% accuracy in the mentioned range, but at $1*10^{20}\ m^{-3}$ can give two times greater Ne, due to pronounce dependence on T. Also, applications is limited to the cases with equal gas and electron temperatures.

3.3. *Programs for comparison of whole profiles*

All published programs (to the authors' knowledge) on electron density determinations via comparison of whole H_β profiles are based on VCS tables. Let us present a brief description of published programs.

3.3.1. *Goode, Davor's EDFIT*

This program (Goode et al, 1984) is based on recalculations of tabulated VCS profiles to $S(\Delta\lambda)$, convolution with Doppler and instrumental profiles. The obtained profiles are normalized on unit intensity and sampled in equally spaced points 0.01nm apart till the 1% of intensity. Intermediate profiles are calculated by interpolation with polinoms. The comparison with experimental data sampled in the same wavelength points, by Ne interval halving algorithm and minimization of sum of square of residuals was performed. The exclusion of points in the center of the profile during the comparison is also included. The results obtained with this program show 2-3 times lower Ne than from H_β halfwidths which are not observed by the comparison of whole profiles by other authors (Thomsen et al, 1991). Also, the reasonable doubts (Chan,Montaser, 1989) that this program uses only profiles with Te=Tg=2500K and that has considerable software mistakes exist.

3.3.2. *Chan, Montaser's NE*

In this program (Chan,Montaser, 1989) intermediate profiles (for nontabulated Ne and Te) are obtained by cubic spline interpolation. Convolution of Doppler and instrumental profiles and area normalization are also performed. Same method as in 3.3.1. for the comparison of resulting profiles with experimental data are used. Authors point out the influence of Te and Tg on Ne determination and errors due to Lagrange polinoms interpolation of profiles.

3.3.3. *Kuraica, Konjević, Platiša, Pantelić FIT_SPC*

In this paper (Kuraica et al, 1992) $S(\Delta\lambda)$ is approximated with parabolic spline and intermediate profiles are obtained by Lagrange polinoms interpolation. This approach enables convolution of Stark and Doppler and instrumental profiles directly by convolution integral via erf functions instead of FFT. The profiles are normalized on unit intensity and superposition of additional profiles is enabled.

3.3.4. *Zhang et all. NNE*

This program (Zhang et al, 1994) is based on same principles as the 3.3.2. but written in C language instead of TURBO BASIC. Furthermore it has possibility to exclude some points in the center of the profile during the comparison with experimental data. Unfortunately authors neither supply as with program source nor explain the data entry.

3.3.5 *Starn et all*

In this case (Starn et al, 1995) normalization on unit intensity (for preventing of background intensity influence and owing to inability to record profile far from the center) was performed. The cubic spline interpolations of profiles and FFT convolution are also used. Determination of Ne via comparison of obtained theoretical profiles and experimental data by Fo instead of usual Ne interval halving is used. The exclusion of points in the center of the theoretical profile is also enabled. Unfortunately this program is written in LabVIEW program language for Machintosh and the author didn't respond to us so, we was unable to test it.

4. CONCLUSION

On the basis of this analysis for estimation of electron density from 10^{20} to 10^{21} m^{-3} we recommend the use of approximate formulas combination i.e. application of Kelleher's

formula for determination of Stark half-half widths and Wiesse's for Ne = f (HWHM) dependence.

The determination of electron density from the whole profiles based on VCS tables, in this concentration range exert systematic error due to inadequate description of central part of the profile caused by non inclusion of ion dynamics effect by unified theory (Kelleher et al, 1973, Cooper et al,1974). The exclusion of some points in the center of the profile at comparison with experimental points can lead to the error greater than 100% (Thomsen et al, 1991). Therefore new calculation and tabulation of hydrogen like profiles, which have to include the influence of ion motion effect are recommended (Griem, 1997). Naimely, tables based on the Model Microfield Method (Stehle, 1994) include this influence and give a better description of the central part of H_β profile than unified theory, but unfortunately the obtained halfwidths (according to author) are greater than experimentally observed and can be used only for estimations of electron densities. All this facts directed our further work towards writing a program for electron density determination based on tables obtained by Monte Carlo simulations.

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