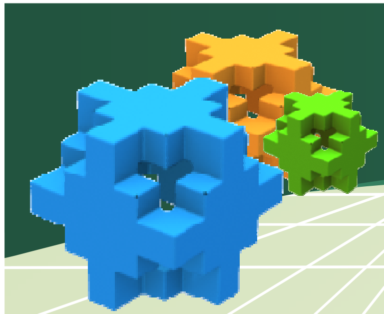

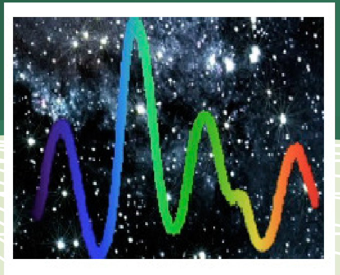

11th Serbia Conference on Spectral Line Shapes in Astropysics
(Sabac, Serbia, August 21-25, 2017)

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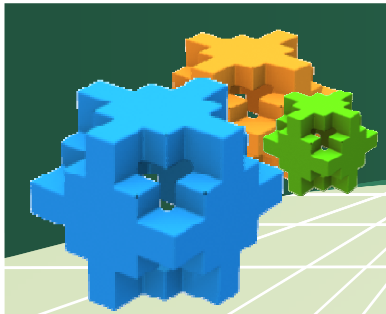
DOPPLER BROADENING OF SPECTRAL LINE SHAPES IN RELATIVISTIC PLASMAS

Objectives



Abstract,

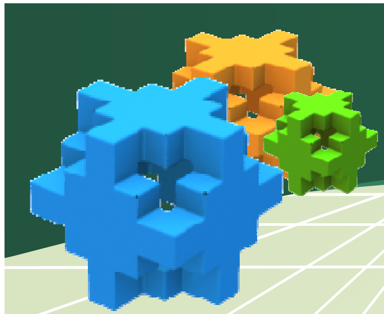
In this work, we report some relativistic effects on the spectral line broadening. In particular, we give a new Doppler broadening in extra hot plasmas that takes into account the possible high velocity of the emitters. This suggests to use an appropriate distribution of the velocities for emitters. We exhibit, an asymmetry in the Doppler line shapes.

Doppler broadening

Classical Doppler broadening: non relativistic case

We assume a motionless observer, looking an emitting atom moves with a velocity ~ $\vec{v}$ *making an angle with the direction x of the observation, records a frequency given by:*

$$\omega(V_x) = \omega_0\left(1 - \frac{V}{c}\cos\theta\right) = \omega_0\left(1 - \frac{V_x}{c}\right)$$



The Doppler line is then obtained by Fourier transform of the delta function averaged with respect Maxwell distribution of velocities:

$$I(\omega) =< \delta(\omega - \omega(V_x)) >= \sqrt{\frac{m}{2\pi k_B T}} \int_{-\infty}^{+\infty} \exp(-\frac{m}{2k_B T} V_x^2)\delta(\omega - \omega(V_x))dV_x$$

$$\sim \int_{-\infty}^{+\infty} \exp(ik\omega)dk \int_{-\infty}^{+\infty} \exp(-\frac{m}{2k_B T} V_x^2 - ik\omega(V_x))dV_x$$

$$\sim \exp\left(-\frac{mc^2}{2k_B T\omega_0^2}(\omega - \omega_0)^2 \right)$$

then the Doppler broadening is given by:

$$\Delta\omega_{Doppler}(Hz) = 7.1574 \times 10^{-7} \times \omega_0 \sqrt{\frac{T}{M}}$$

where M is the emitter mass in atomic mass unit

Relativistic Doppler broadening

When an observer is at a rest, he records the emitted radiation from a moving atom (or ion) with relativistic velocity $\vec{v}$ *, with the frequency equal to :*

$$\omega(\beta) = \omega_0 \gamma (1 + \beta \cos\theta)$$

where γ *and* β *are given by:*

$$\gamma^2 = \frac{1}{1-\beta^2} \qquad \beta = \frac{V}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}}$$

Then :

$$I(\omega) = < \delta(\omega - \omega(\beta)) >_{Juttner} = \int f(\beta) d\beta . \delta(\omega - \omega(\beta))$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int \int \int f(\beta) d\beta \exp(iu(\omega - \omega(\beta))) du \sin\theta d\theta d\phi$$

where $f(\beta)d\beta = \lambda\frac{\gamma^5\beta^2}{K_2[\lambda]}\exp(-\lambda\gamma)d\beta$ *is Juttner distribution and*

$$\lambda = \frac{mc^2}{k\,T}$$

We can write now, the Doppler line to be equal (K2(x) is a Bessel function)

$$I(\omega) = \frac{\lambda}{2\pi K_2(\lambda)}\int_{-\infty}^{+\infty}\int\int\int \gamma^5\beta^2\exp(-\lambda\gamma)d\beta\exp(iu(\omega-\omega(\beta)))du\sin\theta d\theta d\phi$$

As the factor front of the integral not contributes to the width, we can cancel it and replacing ω(β) by its expression, we find:

$$I(\omega) = \int_{-\infty}^{+\infty}\exp(iu\omega)du\int_0^1\gamma^5\beta^2\exp(-\lambda\gamma)\exp(-iu\omega_0\gamma)d\beta\int_0^\pi\exp(iu\omega_0\gamma\beta\cos\theta)\sin\theta d\theta$$

Finally, after integration we get the relativistic Doppler line

Let $\overline{\omega} = \frac{\omega}{\omega_0}$ ***the reduced frequency***

$$I(\hat{\omega}) = \int_1^\infty \gamma d\gamma \exp(-\lambda\gamma)\left(sign(\hat{\omega} - \gamma + \frac{\sqrt{\gamma^2 - 1}}{\gamma}) - sign(\hat{\omega} - \gamma - \frac{\sqrt{\gamma^2 - 1}}{\gamma}) \right)$$

This formulae allows us the following results

T=10^{12} K

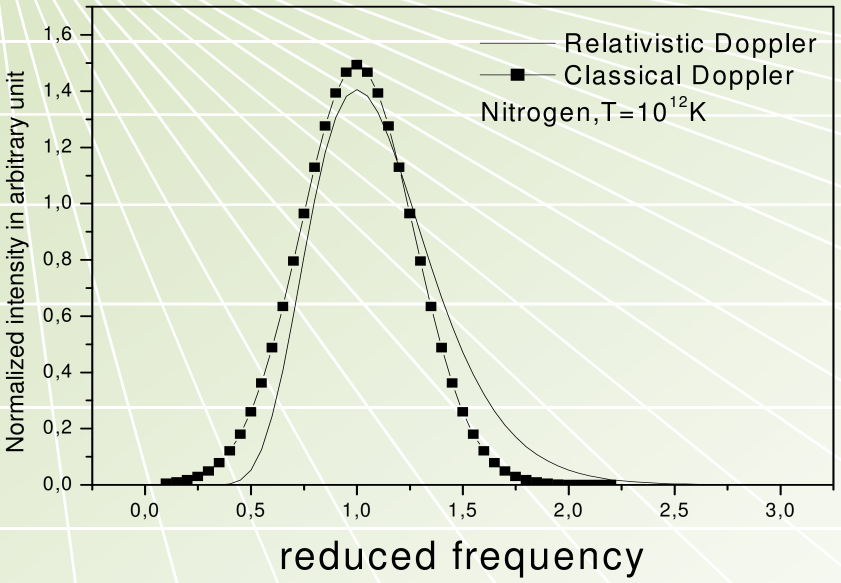


Fig 1: Comparaison between te classical and relativistic Doppler effects for Nitrogen at 10^{12} K

In this figure we remark the asymmetry. No shift is expected. This condition is of early universe (Cosmology, early Universe).

T=10^{11}K

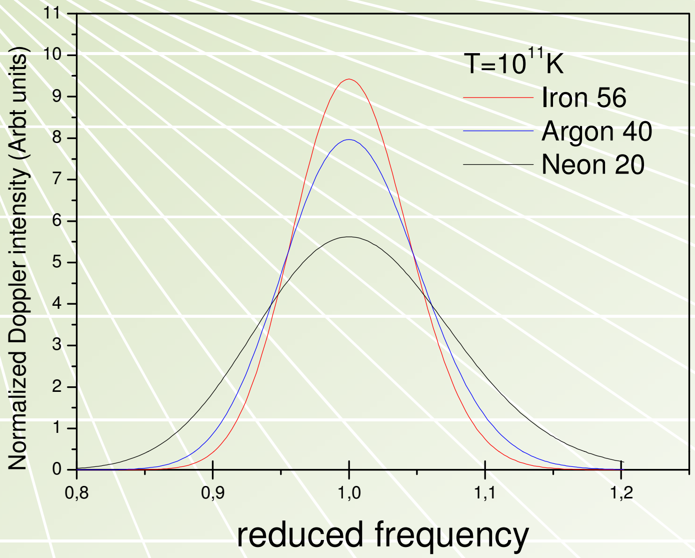


Fig 2: Relativistic Doppler line for different emitters at 10^{11} K

we remark an asymmetry in the blue region for different atoms.
The asymmetry is more important for light atoms

For various temperature we observe

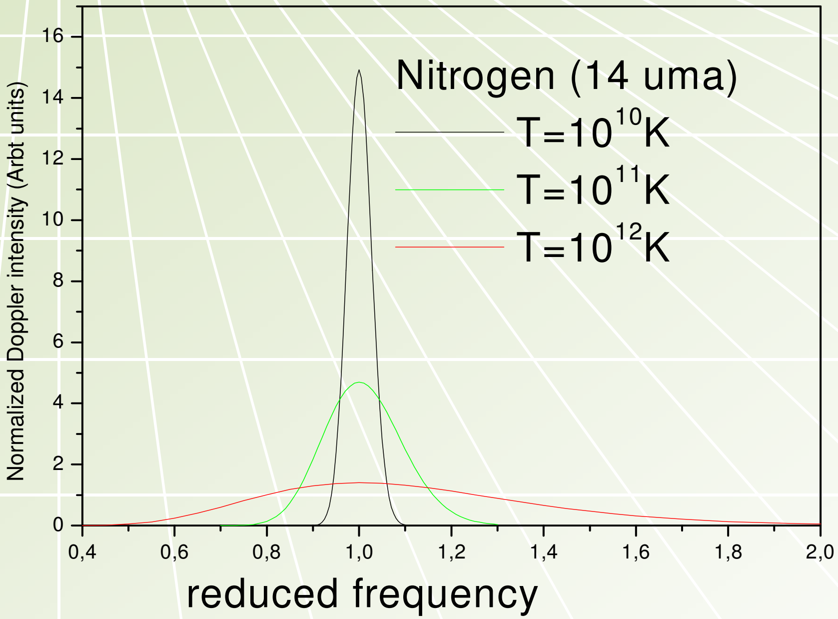


Fig 3: Relativistic Doppler effects for Nitrogen at different temperature

we remark an asymmetry is more important for high temperature

$$Assym = \frac{W_R - W_L}{W_R} \times 100$$

//////////////////////	Fe	Ar	N
$\underline{w}_1$	0.952	0.944	0.906
$\underline{w}_2$	1.050	1.060	1.104
$W_L = 1 - \underline{w}_1$	0.048	0.056	0.094
$W_R = \underline{w}_2 - 1$	0.050	0.060	0.104
$Assym = \frac{W_R - W_L}{W_R} \times 100$	4	7	10

Tab.1: Asymmetry for different masses at 10^{11} K

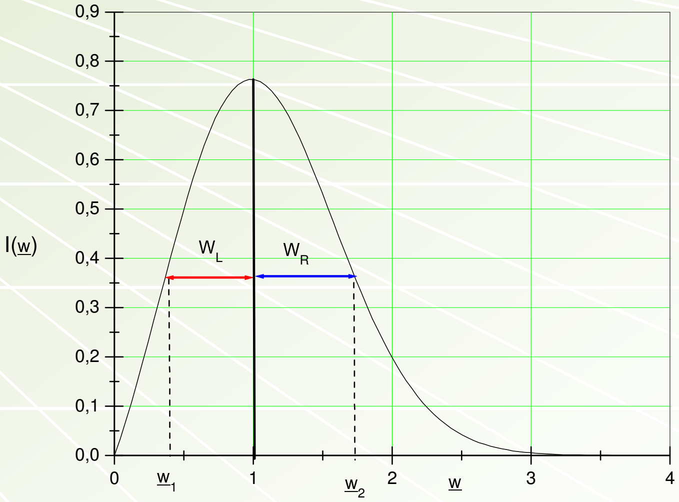


Fig.4: Asymmetry on spectral line

We can see the relationship between mass and asymmetry, so that the light atoms have increased asymmetry

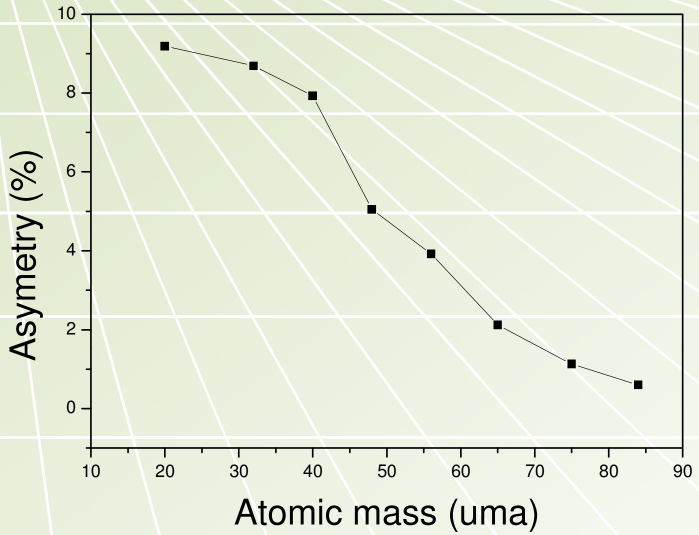


Fig 5: Asymmetry as function of mass of emitter at 10^{11} K

We observe the decreasing of asymmetry with the mass for a high temperature

Reference: **Formulation of relativistic Doppler-broadened absorption line profile,** EPL (Europhysics Letters), Volume 97, Number 2 (2012)

LOGO

THANK YOU!