The radio recombination lines of hydrogen

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Introduction

Radio lines are observed in regions with number densities $10^3 \leq N \leq 10^4$ and $T \simeq 10^4$K. Under these conditions hydrogen is mostly ionized and so free electrons and protons can produce broadening of hydrogen lines emitted in transitions between highly excited states of neutral hydrogen. These are the so called HII regions.

In 1945, van der Hulst was the first astronomer to consider the possibility of radio line radiation between highly excited levels of hydrogen. He made some estimates of the Stark broadening and concluded that such lines were unlikely to be dedected. Other astronomers were similarly pessimistic.

In 1959, Kardashev first predicted that these lines could be observed and gave some estimates for the Doppler and Stark broadening to be expected. His work inspired two Russian groups at Pulkovo and at the Lebedev Institute to carry out observations. They were the first to obtain definitive results for hydrogen radio lines. Results were reported at the XII General Assembly of the IAU in Hamburg in 1964 and subsequently other observatories detected lines.

The actual estimates of the line broadening made by Kardashev proved to be inconsistent with observation. However Hans Griem (1967) realised that under these conditions both proton and electron collisions should be treated using the impact theory of line broadening and did obtain consistent results. He found proton impact to be unimportant. Essentially the same conclusions were reached by me in (1972). Many observations of radio lines emitted from various sources have been published since then.

The book by Gordon and Sorochenko (2009) gives a comprehensive review of observations and theory.

Reasons for reexamination of the line broadening theory

In 2000, Bell *et al* published observations of radio lines emitted at frequencies around 6 GHz and 17.6 GHz by Orion A and W51. More details have been given in their (2011) paper.

The results for 17.6 GHz did not present any surprises but the ones at 6 GHz did.

Transitions $n' = n + \Delta n \to n$ observed were: $(n, \Delta n)$ = (102,1), (129,2), (147,3), (174,5), (184,6), (194,7), (202,8), (210,9), (217,10), (224,11), (230,12), (236,13), (241,14), (247,15), (252,16), (257,17), (261,18), ((266,19), (270,20), (274,21), (278,22), (282,23), (286,24) and (289,25).

Lines above $(n, \Delta n)$ = (202,8) showed unexpected narrowing. Jordan and Alexander (private communication) have made new observations of lines from the Orion Nebula and do not observe this. Hence the recent reexamination of line broadening theory by several authors, Oks (2004), Griem (2005) and Watson (2006). A new analysis of Bell's data has been published by Hey (2013).

In previous calculations complete profiles for the lines have not been calculated. The purpose of the present calculations, see also Peach (2014), is to obtain complete profiles and then extract the widths.

The impact theory of Baranger

Baranger (1958) developed impact theory for the case of overlapping lines, and for an isolated line this reduces to the well-known expression for the full-half width of the Lorentzian profile for the transition $i \rightarrow f$ given by

$$W = \left[Nv \left(\sigma_i(\text{in}) + \sigma_f(\text{in}) + \int \mathrm{d}\Omega |f_i(\Omega) - f_f(\Omega)|^2 \right) \right]_{av}$$

In the case of hydrogen we have to consider all the overlapping components in the $(n, \Delta n)$ transition. For low values of n the third term dominates, but for a fixed value of Δn as n increases the elastic scattering amplitudes f_i and f_f coherently cancel and only the inelastic cross sections $\sigma_i(\text{in})$ and $\sigma_f(\text{in})$ contribute. For overlapping lines in hydrogen the elastic scattering terms should be interpreted as including all transitions for which $\Delta E = 0$.

The theory of Baranger leads to the following formal expression for the line profile $L(\omega)$:

$$L(\omega) = \frac{1}{\pi}\mathcal{R}\sum << n_i l_i (n_f l_f)^* ||\delta|| n_i l_i' (n_f l_f')^* >>$$
$$\times << n_i l_i' (n_f l_f')^* ||[\mathbf{h} - \mathsf{i}(\omega - \omega_0)]^{-1}|| n_i l_i (n_f l_f)^* >> ,$$

where $(\omega - \omega_0)$ is the angular frequency separation from the centre of the line. The matrix elements are in reduced line space and $\mathcal{R}$ denotes 'real part of'. This has been used as the basis for the calculation of the complete line profile.

In order to calculate the average over all possible collisions we need the Maxwell distribution $f(v)$ for the relative velocity v which is defined by

$$f(v) = 4\pi v^2 \left(\frac{M}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{Mv^2}{2kT}\right) \; ; \quad \int_0^\infty f(v)\,\mathrm{d}v = 1\,,$$

where T is the temperature and M is the reduced mass of the emitter plus perturber.

Semi-classical impact parameter treatment

The dominant interaction between the emitting hydrogen atom and the electron/proton perturber is given by

$$V(\mathbf{r},\mathbf{R}) = \pm \mathrm{e}^2 \frac{\mathbf{r}\cdot\mathbf{R}}{R^3}\,; \qquad \mathbf{R} = \rho + \mathbf{v}t\,; \quad \rho\cdot\mathbf{v} = 0\,,$$

where $\mathbf{R}$ gives the position of the perturber relative to the emitter and it is assumed that the perturber follows a straight-line path where ρ is the impact parameter and t is the time. This expression for $V(\mathbf{r},\mathbf{R})$ gives the leading term in the interaction provided that $R > r$.

Second-order time-dependent perturbation theory is used to calculate the cross sections for the emitter-perturber collisions. The matrix elements of **h** are given by

$$<< n_i l_i'(n_f l_f')^* ||\mathbf{h}|| n_i l_i (n_f l_f)^* >>= N \int_0^\infty v f(v)\,\mathrm{d}v$$
$$\times \left\{ \frac{1}{2} \left[\sum_k \sigma_{ik}(v) + \sum_k \sigma_{fk}(v) \right] \delta_{ii'} \delta_{ff'} - \sigma_{i'f'if}(v) \right\} ,$$

where $\sigma_{jk}(v); j = i, f$ is the cross section for the transition $n_j l_j \rightarrow n_k l_k$ and $\sigma_{i'f'if}(v)$ is a 'cross section' arising from a mixed term. It is assumed that the number densities N of the electron and proton perturbers are the same and that they each have the same kinetic temperature T.

When only the dipole interaction is included the line profile is symmetric and unshifted from the line centre.

Inevitably, some uncertainty is introduced by the necessity of using cutoff parameters when evaluating the integrals over the impact parameter ρ. The procedure adopted here is that the collisions are split up into two types, strong or weak. For scattering by the emitter in state nl we define the mean radius $\bar{r}_{nl}$ of the nl state and collisions are strong if $\rho < \bar{r}_{nl}$. The conditions given by Seaton (1962) for evaluating the cross sections have been used in the present calculations for both electron and proton impact. The integrals for which $\Delta E = 0$ diverge logarithmly for large values of ρ, and so the upper cutoff for ρ is varied to test the sensitivity of the width to its choice. Two choices are made; $\rho_{max} \equiv \rho_D$ where ρ_D is the Debye radius, or $\rho_{max} \equiv \rho_n$ where ρ_n is the nearest neighour distance.

References

M. Baranger, Phys. Rev. **112** 855 (1958)

M.B. Bell *et al*, Publ. Astron. Soc. Pac. **112** 1236 (2000)

M.B. Bell *et al*, Astrophys. Space. Sci **333** 377 (2011)

M.A. Gordon and R.L. Sorochenko, 'Radio Recombination Lines', Springer, NY (2009)

H.R. Griem, Astrophys. J. **148** 547 (1967); **620** L133 (2005)

J.D. Hey, J. Phys. B: At. Mol. Opt. Phys. **46** 175702 (2013)

J. Alexander and S. Gulyaev, private communication

E. Oks, Astrophys. J. **609** L25 (2004)

G. Peach, Astrophys. Lett. **10** 129 (1972)

G. Peach, Adv. Space Research **54** 1180 (2014)

M.J. Seaton, Proc. Phys. Soc. **79** 1105 (1962)

J.K.G. Watson, J. Phys. B: At. Mol. Opt. Phys. **39** 1889 (2006)

Results

Hey (2013) has carried out a new analysis of Bell's data taking into account relative intensities of the lines as well as their widths and these are shown in the tables. His results are compared with the present results which take into account both electron and proton broadening and using both Debye and nearest neighbour cutoffs for ρ_{max}.

Orion A. Analysis by Hey (2013) of Bell's observations for FWHM impact widths $W_h(e)$ in MHz compared with present results, $W_p(e)$ and $W_p(ep)$ where $p = D, n$. N and T are in units of 10^3cm^{-3} and 10^4 K respectively.

Line	N	T	$W_h(e)$	$W_D(e)$	$W_n(e)$	$W_D(ep)$	$W_n(ep)$
(102,1)	7.75	1.300	0.095	0.078	0.078	0.087	0.084
(129,2)	6.12	1.296	0.186	0.152	0.152	0.198	0.178
(147,3)	4.34	1.276	0.216	0.180	0.178	0.269	0.225
(174,5)	7.36	1.301	0.696	0.590	0.580	1.110	0.827
(184,6)	4.56	1.253	0.537	0.463	0.452	0.983	0.693
(202,8)	3.59	1.224	0.612	0.540	0.522	1.397	0.898
(252,16)	0.584	1.228	0.244	0.234	0.216	1.117	0.577
(274,21)	0.416	1.146	0.254	0.256	0.228	1.570	0.748

W51. Analysis by Hey (2013) of Bell's observations for FWHM impact widths $W_h(e)$ in MHz compared with present results, $W_p(e)$ and $W_p(ep)$ where $p = D, n$. N and T are in units of 10^3cm^{-3} and 10^4 K respectively.

Line	N	T	$W_h(e)$	$W_D(e)$	$W_n(e)$	$W_D(ep)$	$W_n(ep)$
(102,1)	2.62	2.094	0.030	0.024	0.024	0.028	0.027
(129,2)	2.34	2.031	0.066	0.052	0.052	0.070	0.063
(147,3)	1.53	1.994	0.070	0.057	0.056	0.087	0.074
(174,5)	2.94	1.994	0.255	0.209	0.206	0.405	0.308
(184,6)	2.37	1.933	0.256	0.213	0.208	0.461	0.332
(194,7)	2.62	1.952	0.345	0.291	0.283	0.690	0.475
(202,8)	2.36	1.948	0.363	0.310	0.299	0.808	0.533
(210,9)	1.51	1.914	0.272	0.234	0.225	0.673	0.428
(224,11)	1.29	1.920	0.300	0.263	0.250	0.885	0.526

Conclusions

a) For a fixed value of Δn, only the contributions from collisions with $\Delta E \neq 0$ contribute significantly as n increases. This is because there is increasing cancellation between the upper and lower levels, n_i and n_f, of the contributions from collisions with $\Delta E = 0$.

b) Proton impact is not effective for $\Delta n \neq 0$ until large values of n are reached, but dominates for $\Delta E = 0$.

c) Hence for sufficiently large values of n condition a) holds for both electron and proton impact.

d) However, for lines within a narrow frequency band the cancellation between contributions to the width from collisions with $\Delta E = 0$ becomes less severe and proton impact starts to make a significant contribution.