POLARIZATION IN SPECTRAL LINES FORMED BY SCATTERING: FROM THE LINE CENTER TO THE FAR WINGS

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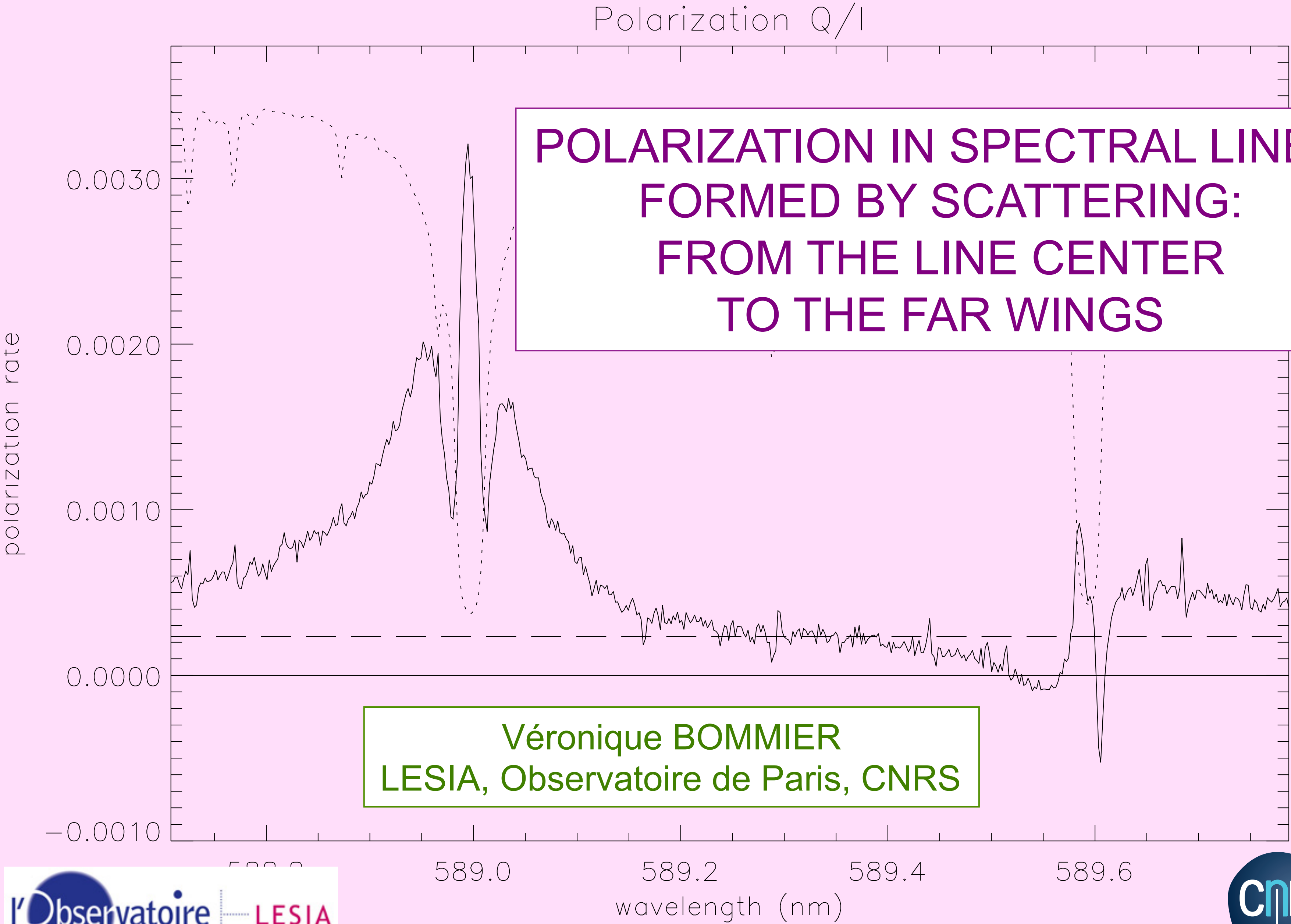


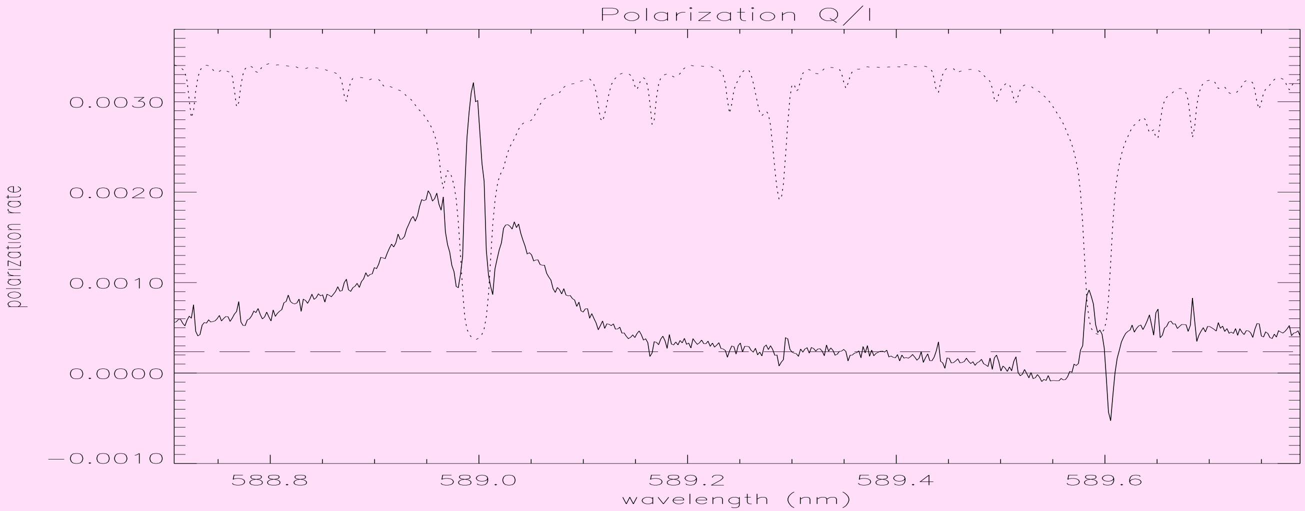
10th SCSLSA, Srebrno jezero, 15-19 June 2015

Objective

– Modeling the line formation **including polarization**
  Magnetic field, multilevel-multiline, polarization profile, far wings
  Solving the coupled statistical equilibrium and radiative transfer equations, for the polarized atom

– Interpreting the **Second Solar Spectrum** (Stenflo & Keller, 1997)
  Linear polarization formed by scattering and observed inside the solar limb

Na I D lines



– 30% of the lines display a M-type polarization profile
  Belluzzi & Landi Degl'Innocenti, 2009, A&A 495, 577, & Belluzzi's PhD

Going out of the 2-level approximation

is solving the system of statistical equilibrium equations (SEE)

But how taking into account the partial redistribution (PRD) ?

The SEE accounts for

- absorption
- emission

But how taking into account

- absorption *followed* by emission ?
- how the system may have "memory" ?

Answer:

- by going out of the "short-memory" approximation
- i.e., by overcoming the Markov approximation
  (Bommier, 1997, A&A 328, 706 & 726)

This will also help for line profiles in SEE

The Markov approximation

or short-memory approximation

Hamiltonian atom+radiation: $H = H_0 + V$

Schrödinger equation in interaction representation: $\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t) = \left[\tilde{V}(t), \tilde{\rho}(t)\right]$

which can be integrated in: $\tilde{\rho}(t) = \tilde{\rho}(0) + \frac{1}{\mathrm{i}\hbar}\int_0^t \left[\tilde{V}(t-\tau), \tilde{\rho}(t-\tau)\right]\mathrm{d}\tau$

Markov approximation: $\tilde{\rho}(t) = \tilde{\rho}(0) + \frac{1}{\mathrm{i}\hbar}\int_0^t \left[\tilde{V}(t-\tau), \tilde{\rho}(t)\right]\mathrm{d}\tau$

– Physical meaning: ρ does not keep memory of his past history during the process

Validity: the characteristic ρ evolution time Γ >> the interaction correlation time τ_c
Cohen-Tannoudji (1975): the validity condition is fulfilled for weak radiation field

Consequence: the ρ finite life-time (inverse of Γ) is not taken into account in the process
the line width, or profile, is discarded from the formalism
at its place, one has

$$\int_0^{+\infty} \mathrm{e}^{-(\omega-\omega_0)\tau}\mathrm{d}\tau = \frac{1}{2}\delta(\omega-\omega_0) + \mathrm{i}P(\omega-\omega_0)$$

P : Cauchy Principal Value

Getting out of the Markov approximation

The Markov approximation intervenes in a perturbation development

Reporting the integral equation in the differential one

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\mathrm{i}\hbar}\left[\tilde{V}(t),\tilde{\rho}(0)\right]-\frac{1}{\hbar^2}\int_0^t\left[\tilde{V}(t)\left[\tilde{V}(t-\tau),\tilde{\rho}(t-\tau)\right]\right]\mathrm{d}\tau$$

Markov approximation closes the development:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\mathrm{i}\hbar}\left[\tilde{V}(t),\tilde{\rho}(0)\right]-\frac{1}{\hbar^2}\int_0^t\left[\tilde{V}(t)\left[\tilde{V}(t-\tau),\tilde{\rho}(t)\right]\right]\mathrm{d}\tau$$

Getting out of the Markov approximation is pursuing the perturbation development

at order-4:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\hbar^4}\int_0^t\mathrm{d}\tau\int_0^{t-\tau_1}\mathrm{d}\tau\int_0^{t-\tau_1-\tau_2}\mathrm{d}\tau\left[\tilde{V}(t),\left[\tilde{V}(t-\tau_1),\left[\tilde{V}(t-\tau_1-\tau_2),\left[\tilde{V}(t-\tau_1-\tau_2-\tau_3),\tilde{\rho}(t-\tau_1-\tau_2-\tau_3)\right]\right]\right]\right]$$

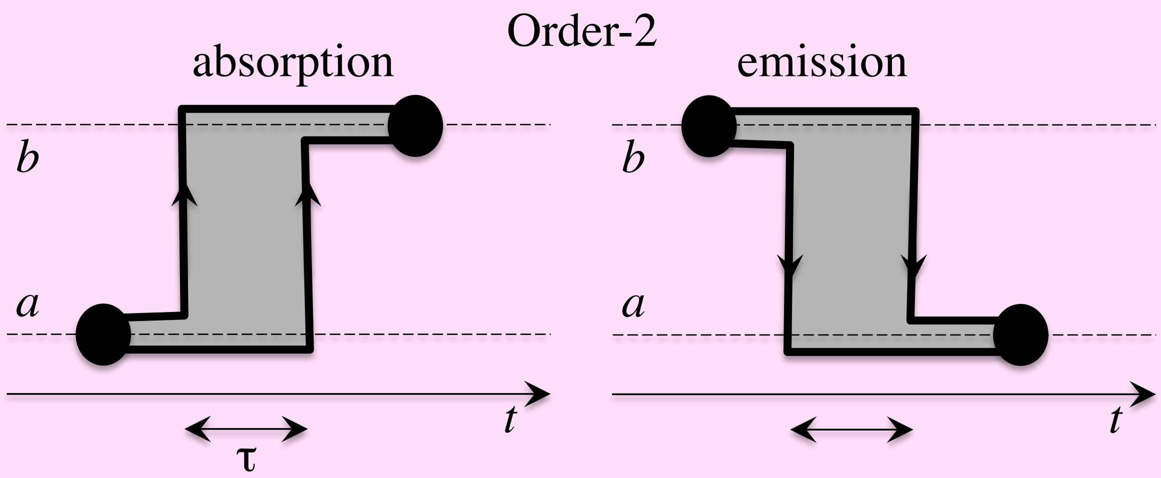
Markov approximation closes again the development

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t)=\frac{1}{\hbar^4}\int_0^t\mathrm{d}\tau\int_0^{t-\tau_1}\mathrm{d}\tau\int_0^{t-\tau_1-\tau_2}\mathrm{d}\tau\left[\tilde{V}(t),\left[\tilde{V}(t-\tau_1),\left[\tilde{V}(t-\tau_1-\tau_2),\left[\tilde{V}(t-\tau_1-\tau_2-\tau_3),\tilde{\rho}(t)\right]\right]\right]\right]$$

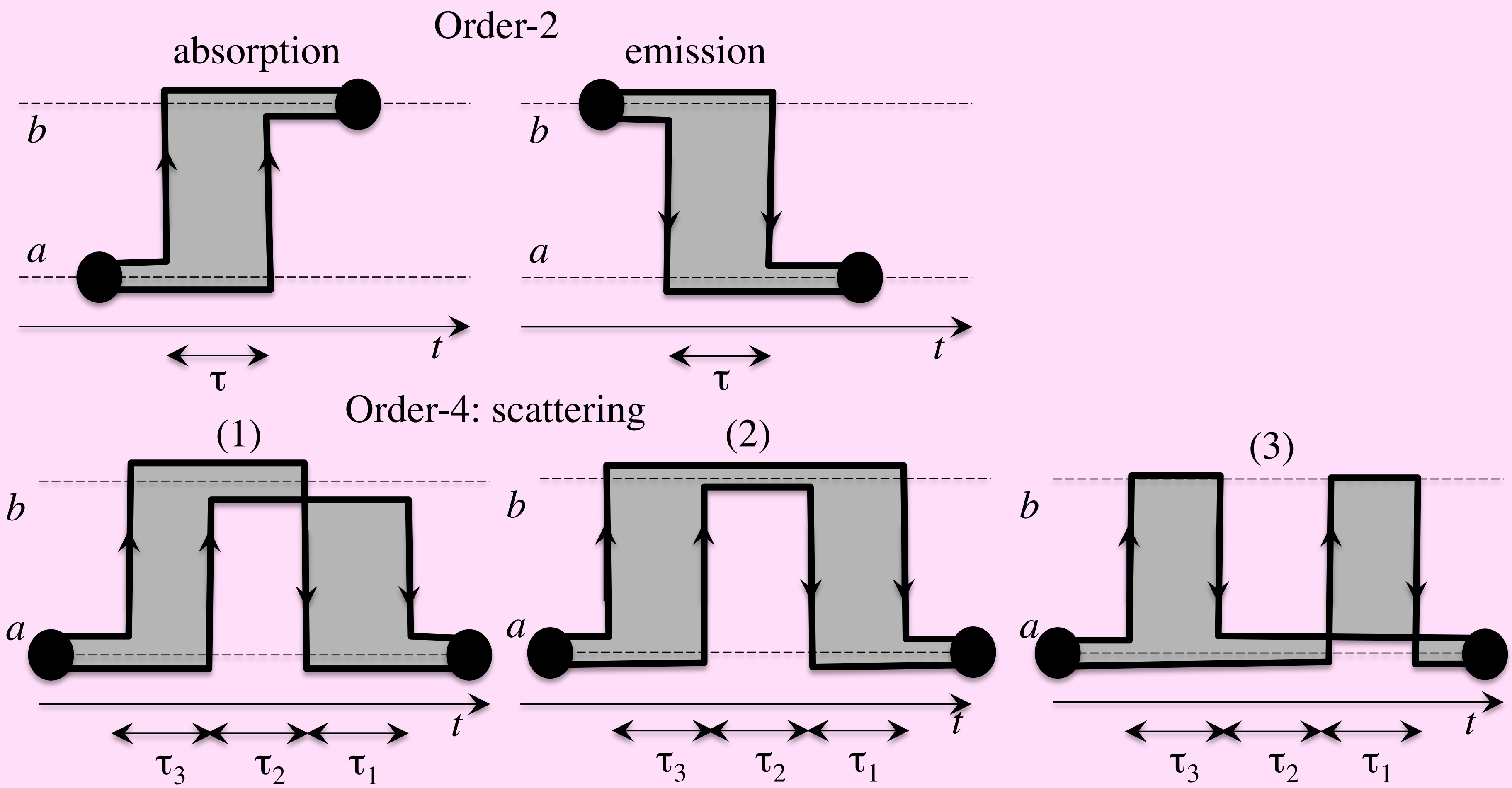
and so on.

and so on…. ➔ series development ➔ what is the limit ?

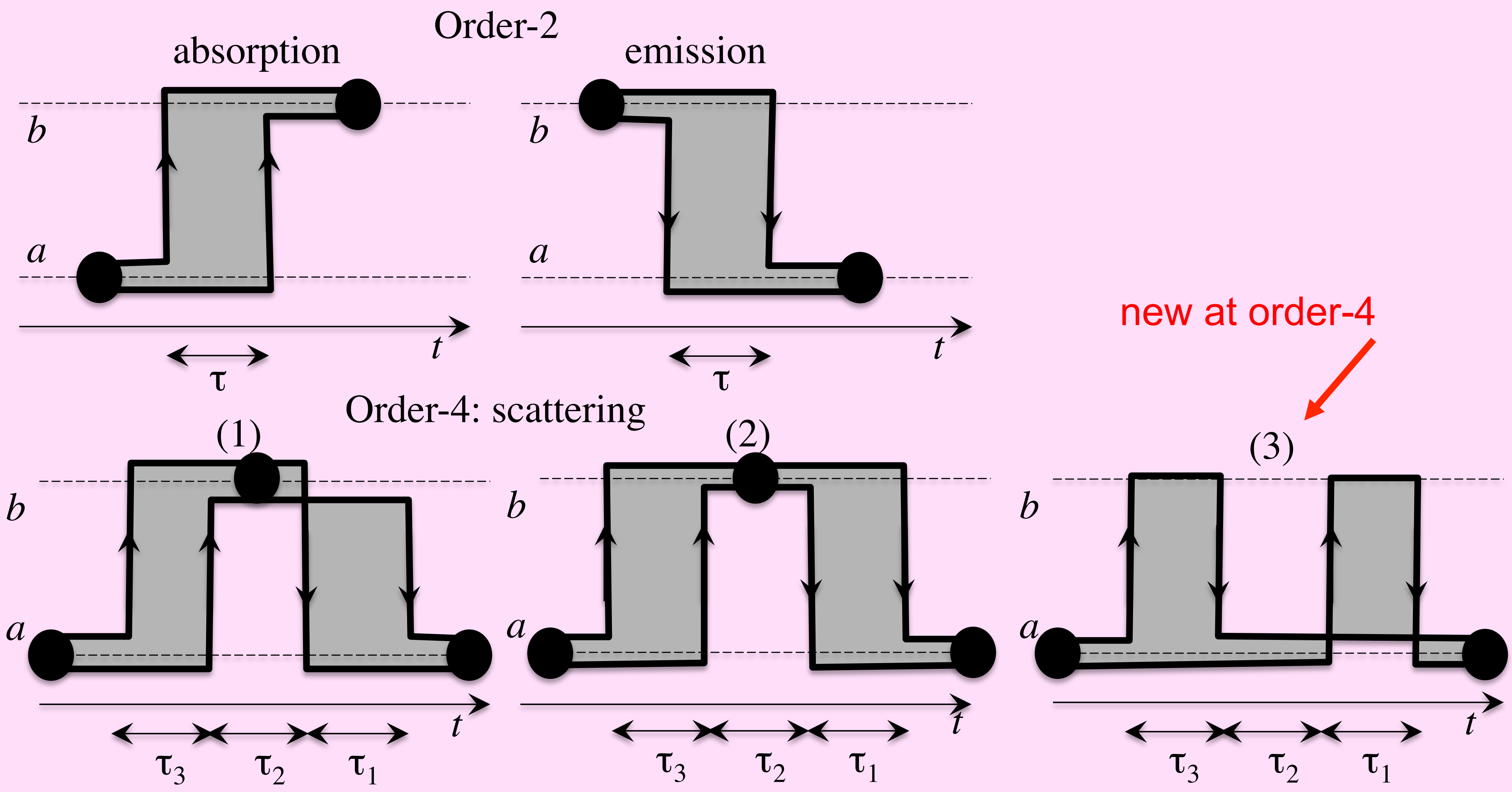
Transforming the series development into a summation



Transforming the series development into a summation

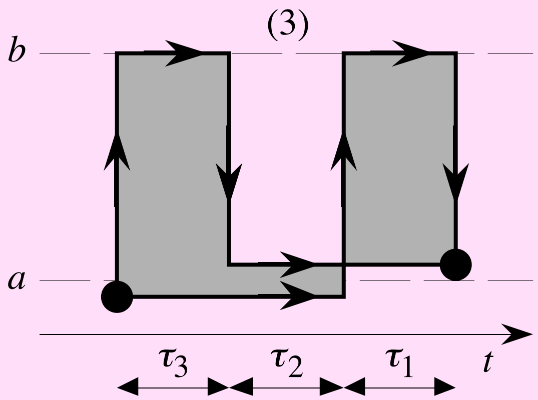


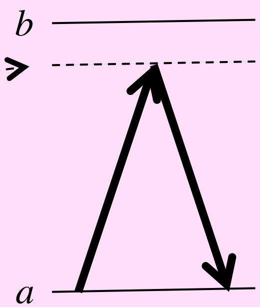
Transforming the series development into a summation



the new term at order-4

A new process appears at order-4, which can be represented as:





The two transition amplitude do not stay at the same time in the upper level b

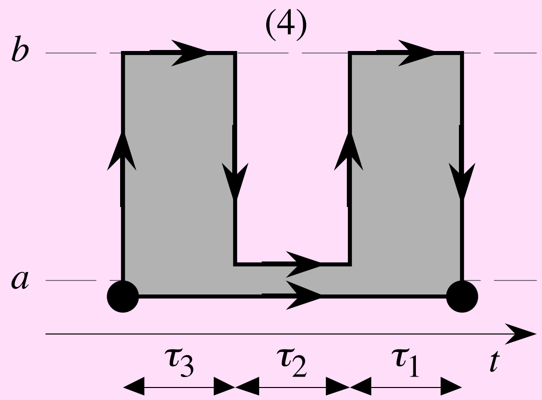
The b level is « never populated », or « virtual »
There is no absorption, nor emission
There is only scattering, with frequency conservation
This is Rayleigh scattering (can be generalized to Raman scattering)
This intervenes in the far wings

There is frequency coherence between the « absorbed » and the « emitted » photons,
Such a coherence which is rendered impossible by the Markov approximation

2^{nd} new term at order-4 in the emissivity

The other new processes broaden the line

new broadening process at order-4:



Resummation

The statistical equilibrium equation remains the same as usual, except that in place of the δ function, at the profile place, appears a quantity of the generic form

Perturbation development manually written

$$\varphi\left\{1-\frac{A_{ba}}{2}\varphi+\frac{A_{ba}^{2}}{2^{2}}\varphi^{2}-\frac{A_{ba}^{3}}{2^{3}}\varphi^{3}+\dots\right\}$$

One sees that it behaves as

$$\varphi\left\{\sum_{n=0}^{\infty}\left[-\frac{A_{ba}}{2}\varphi\right]^{n}\right\}$$

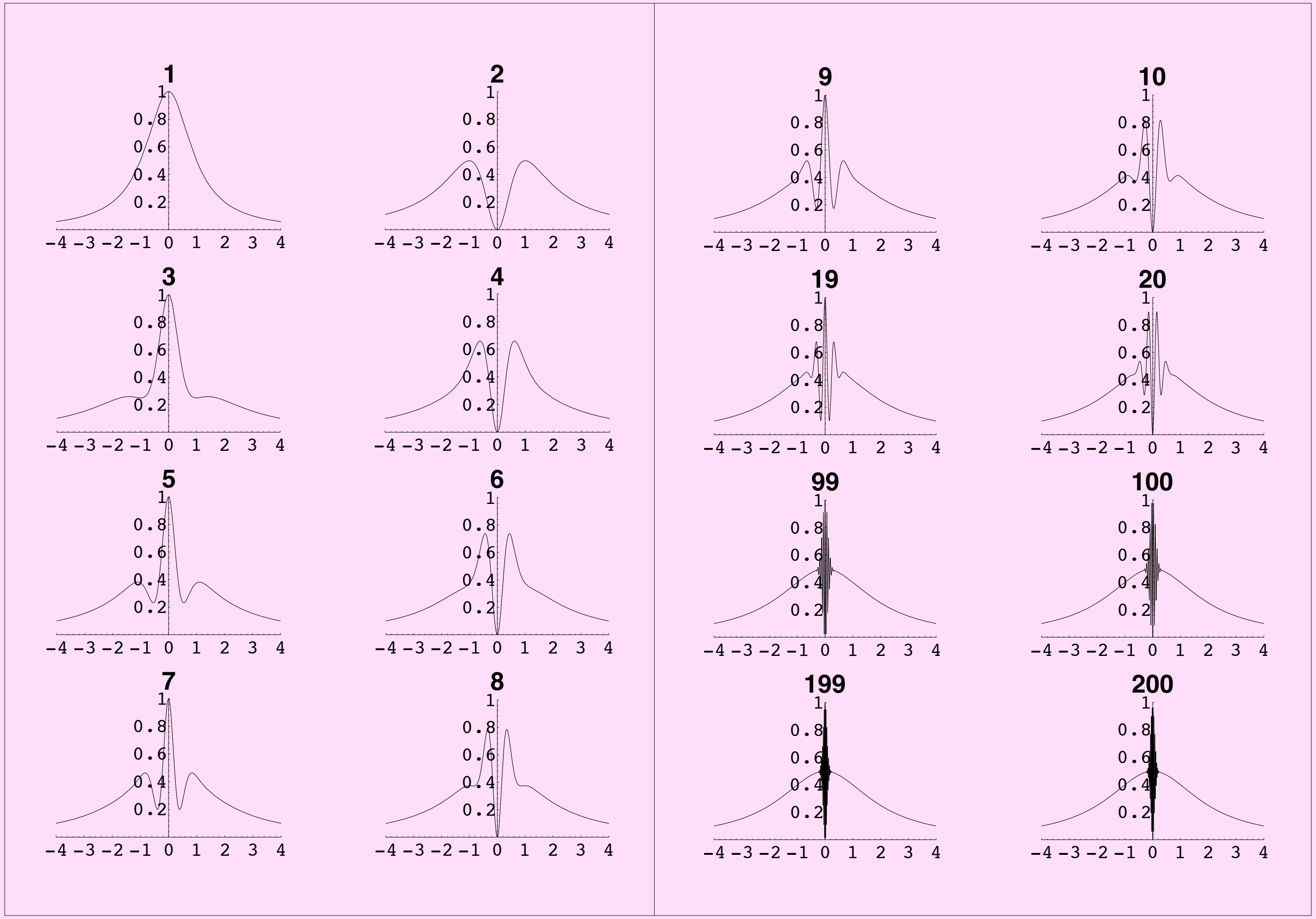
which can be resummed in

$$\frac{\varphi}{1+\frac{A_{ba}}{2}\varphi}$$

which introduces A_{ba} as a half-half-width in the profile

The resummed theory is non-perturbative

Visualization of the resummation effect



Line profiles

impact approximation (Baranger, 1958, Phys. Rev. 111, 494)

in the atomic rest frame, Lorentz profile

$$\tfrac{1}{2}\Phi_{ba}(\nu_0-\nu)=\frac{1}{\gamma_{ba}-\mathrm{i}(\omega_0-\omega+\Delta_{ba})}$$

with

$$\gamma_{ba}=\gamma_{ba}^{(c)}+\tfrac{1}{2}(\Gamma_a+\Gamma_b)$$

and for the collisional part

$$\gamma_{ba}^{(c)}+\mathrm{i}\Delta_{ba}=\left\{1-\langle a|S|a\rangle\langle b|S|b\rangle^*\right\}_{\mathrm{AV}}$$

and when $b=a$

$$\gamma_{aa}=\gamma_{aa}^{(c)}+\Gamma_a \text{ and } \gamma_{aa}^{(c)}=\left\{1-\left|\langle a|S|a\rangle\right|^2\right\}_{\mathrm{AV}}$$

see also Sahal-Bréchot & Bommier, 9th SCSLSA, 2014, JASR, 54, 1164

– each Zeeman component is centered at its exact wavelength (half-sum of 2 profiles for Zeeman atomic coherences)
– the width is assumed to be of same for all Zeeman components

2^{nd} effect: new term at order-4 in the emissivity

$\varepsilon =$

order-2 $\quad \dfrac{h\nu}{4\pi}\dfrac{\nu^3}{\nu_0^3} N\rho_{bb} A_{ba}\phi_{ba}\left(\nu_0-\nu\right)$

$$+\frac{h\nu}{4\pi}\frac{\nu^3}{\nu_0^3} N\rho_{aa} B_{ab}\int \mathrm{d}\nu_1 J\left(\nu_1\right)$$

order-4

$$\left[\frac{1}{2}\Phi^*_{ba}\left(\nu_0-\nu\right)\frac{A_{ba}}{2}\Phi_{aa}\left(\nu-\nu_1\right)\Phi_{ba}\left(\nu_0-\nu_1\right)\right]$$

$\Phi_{ba}\left(\nu_0-\nu_1\right)$: complex profile of half-half-width γ_{ba}

$\Phi_{aa}\left(\nu-\nu_1\right)$: complex profile of half-half-width the lower level a life-time

infinitely sharp lower level a:

$$\left[\frac{1}{2}\Phi^*_{ba}\left(\nu_0-\nu\right)\frac{A_{ba}}{2}\Phi_{a}\left(\nu-\nu_1\right)\Phi_{ba}\left(\nu_0-\nu_1\right)\right]=\frac{A_{ba}}{2\gamma_{ba}}\left\{\delta\left(\nu-\nu_1\right)\phi_{ba}\left(\nu_0-\nu_1\right)-\phi_{ba}\left(\nu_0-\nu\right)\phi_{ba}\left(\nu_0-\nu_1\right)\right\}$$

The order-4 term in the emissivity:

– its integral over one or the other of the frequencies is zero
– it redistributes the frequencies inside the emission profile
– the result is a decoupling between atom and radiation

2-level atom: Redistribution Function

Analytical solution of the statistical equilibrium reported in the emissivity

Γ_R : radiative inverse life-time

Γ_I : inelastic collisions ($b \leftrightarrow a$) inverse life-time

Γ_E : elastic collisions (in b) inverse life-time

$$\gamma_{ba} = \frac{1}{2}\left(\Gamma_R + \Gamma_I + \Gamma_E\right)$$

$$\varepsilon = \frac{h\nu}{4\pi}\frac{\nu^3}{\nu_0^3} N \rho_{aa} B_{ab} \int \mathrm{d}\nu_1 J(\nu_1) \left\{ \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E}\delta(\nu - \nu_1)\phi_{ba}(\nu_0 - \nu_1) + \frac{\Gamma_R}{\Gamma_R + \Gamma_I}\frac{\Gamma_E}{\Gamma_R + \Gamma_I + \Gamma_E}\phi_{ba}(\nu_0 - \nu)\phi_{ba}(\nu_0 - \nu_1) \right\}$$

with polarization:

$$\varepsilon = \frac{h\nu}{4\pi}\frac{\nu^3}{\nu_0^3} N \rho_{aa} B_{ab} \int \mathrm{d}\nu_1 \oint \frac{\mathrm{d}\vec{\Omega}_1}{4\pi} \sum_K \left[w_{J'J}^{(K)}\right]^2 \sum_{j=0}^{3} \left[P_R^{(K)}\left(\vec{\Omega}, \vec{\Omega}_1\right)\right]_{ij} S_j\left(\nu_1, \vec{\Omega}_1\right) \qquad \text{Rayleigh phase matrix}$$

$$\left\{ \frac{\Gamma_R}{\Gamma_R + \Gamma_I + \Gamma_E}\delta(\nu - \nu_1)\phi_{ba}(\nu_0 - \nu_1) + \frac{\Gamma_R}{\Gamma_R + \Gamma_I + D^{(K)}}\frac{\Gamma_E - D^{(K)}}{\Gamma_R + \Gamma_I + \Gamma_E}\phi_{ba}(\nu_0 - \nu)\phi_{ba}(\nu_0 - \nu_1) \right\}$$

The collision rates weight the contributions of the different redistribution types
(coherent or complete)

Bibliography

– this theory, for a 2-level atom, with polarization and magnetic field
Bommier, V., 1997, A&A, 328, 706 & 726
+ Bommier, V., 1999, ASSL 243 (SPW2), 43
for Raman scattering and Doppler redistribution
(the statistical equilibrium has to be solved for each velocity class of the atoms)

– full agreement about the redistribution functions
and the physical description of the Rayleigh scattering
with Omont, Smith, Cooper, 1972, ApJ, 175, 185

– previous papers make use of the emissivity developed in two terms,
but from empirical derivation
Hubeny, Oxenius, Simonneau, 1983, JQSRT, 29, 495
Hubeny, I., 1985, Bull. Astron. Inst. Czechosl., 36, 1
Hubeny & Lites, 1995, ApJ, 455, 376
Uitenbroek, H., 1989, A&A, 213, 360

Doppler Redistribution

from Sahal-Bréchot, Bommier & Feautrier, 1998, A&A, 340, 579

Atomic velocity: *external freedom degree*

translational hamiltonian $H_{\text{tr}} = \dfrac{\vec{p}^{\,2}}{2m}$, with eigenstates = plane waves $|\vec{p}\rangle$

General atomic density matrix element $\langle \alpha JM, \vec{p} | \sigma(t) | \alpha' J' M', \vec{p}' \rangle$

Velocity-changing collisions: negligible in the solar atmosphere $\Rightarrow \vec{p}' = \vec{p}$

Then $\langle \alpha JM, \vec{p} | \sigma(t) | \alpha' J' M', \vec{p} \rangle = \langle \alpha JM | \sigma(\vec{v}, t) | \alpha' J' M' \rangle$

Normalization by the atomic velocity distribution function $\sigma(\vec{v}, t) = f(\vec{v}) \rho(\vec{v}, t)$

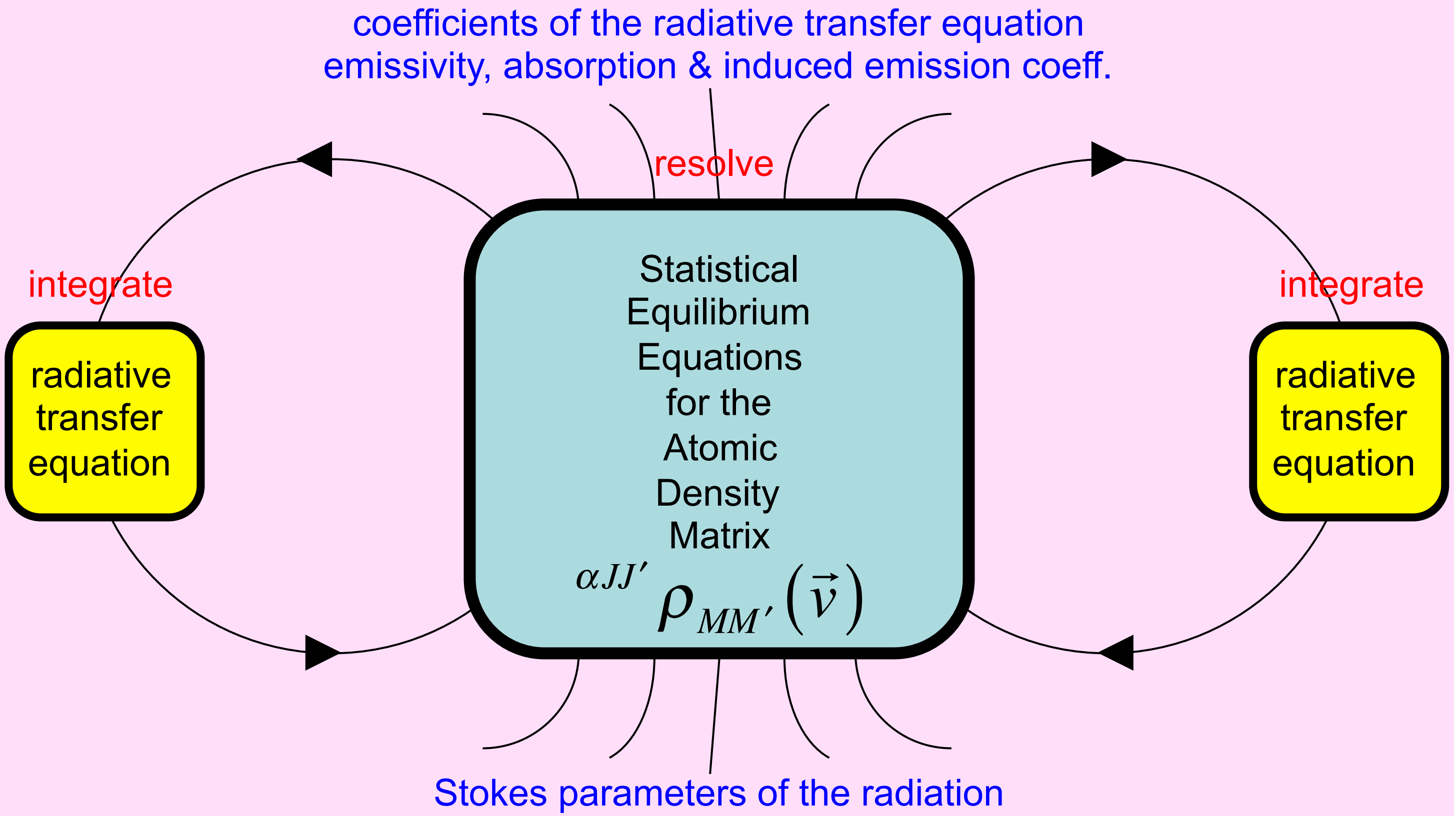
Doppler effect on frequencies $\tilde{\nu} = \nu \left(1 - \dfrac{\vec{\Omega} \cdot \vec{v}}{c} \right)$

New!

The statistical equilibrium equations (SEE) have to be resolved ***for each velocity class***

XTAT, a code based on this theory for modeling the polarized line formation

Centered on statistical equilibrium resolution for the multilevel atom
(iterative method)

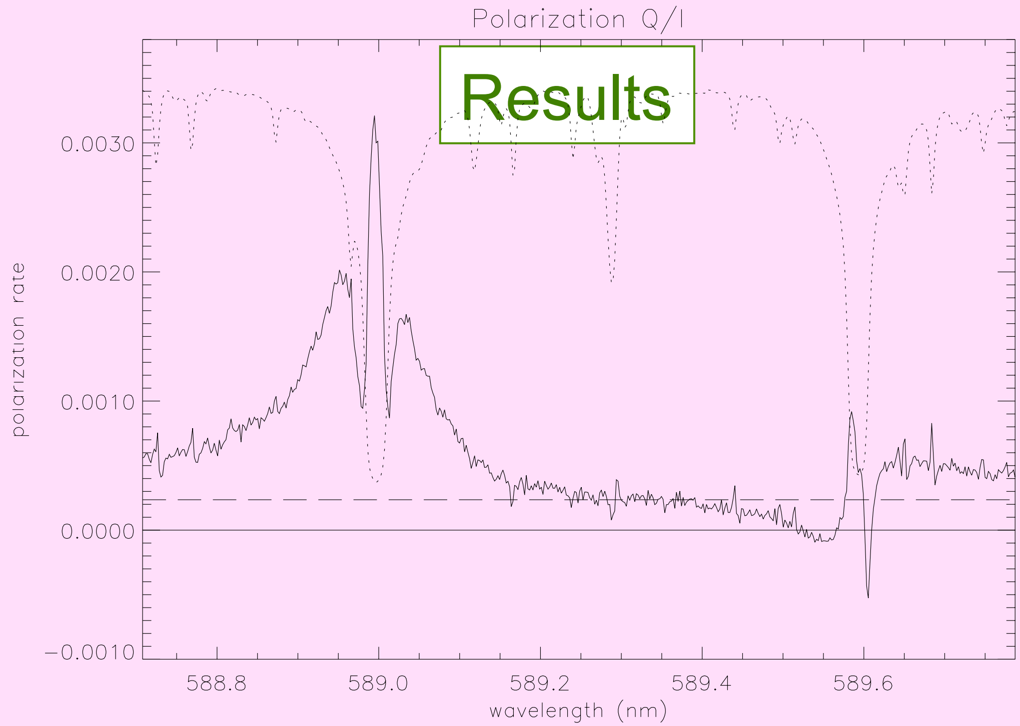


hybrid MPI-OpenMP PARALLELIZED

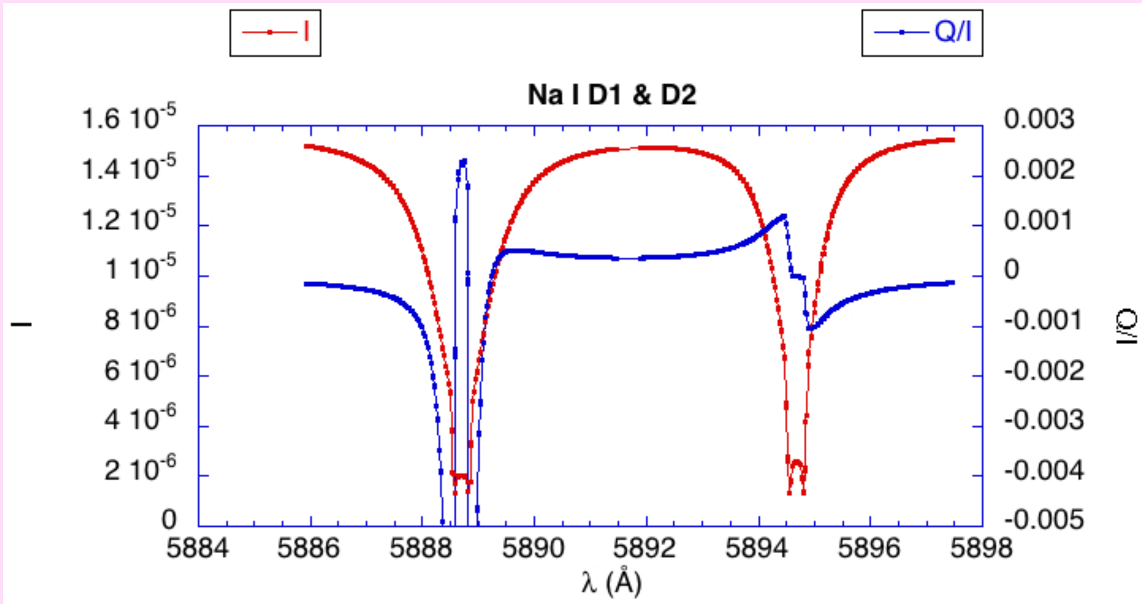
Specificities

- No redistribution functions,
  replaced by the 2nd term of the emissivity stemmed from the order-4 of the development
- Radiative transfer integration: short characteristics method
  thanks to Ibgui et al., 2013, A&A, 549, A126 for a new cubic short characteristics method
- Numerical integrations (radiation and velocities): Gauss methods
  - 256 depths
  - 6 velocity moduli
  - 2 inclinations
  - 4 azimuths
- Initialization: "ultracold" atom or LTE (Boltzmann)
- Ionization: Saha equilibrium is assumed
- Solar atmosphere model: Maltby et al. (1986), close to VAL-C
- Collision rates:
  - elastic (or quasi-elastic) $\mathrm{Na} + \mathrm{H}$
    Kerkeni & Bommier (2002)
  - inelastic $\mathrm{Na} + e^{-}$
    semi-classical perturbation method of Sahal-Brechot (1969)
- Acceleration (Ng, preconditioning) studied but found inefficient
- Hybrid MPI-OpenMP parallelized: 16384 threads running simultaneously
- Machine: IBM Blue Gene/Q (IDRIS, Orsay, France)

Results

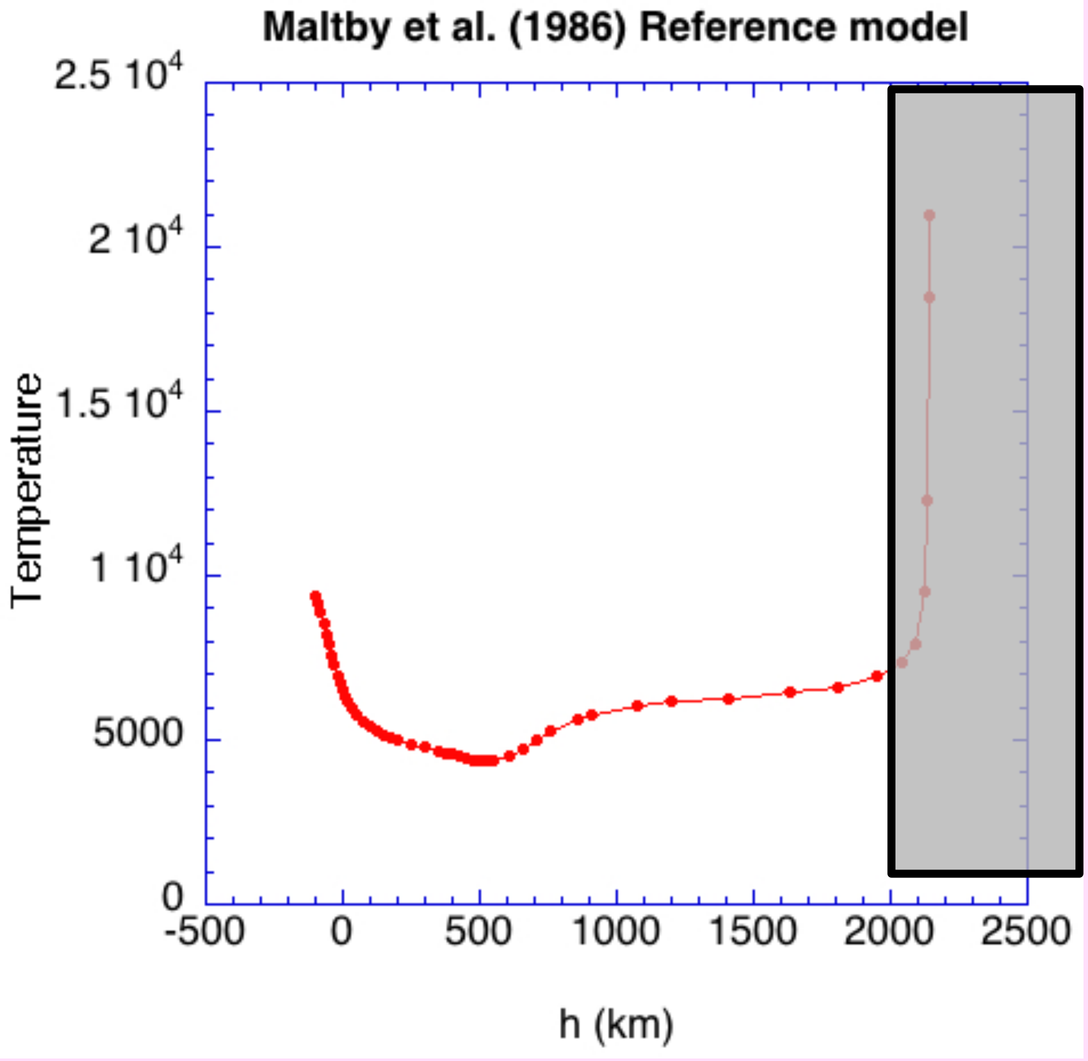


Polarization Q/I
polarization rate
0.0030
0.0020
0.0010
0.0000
−0.0010
588.8
589.0
589.2
589.4
589.6
wavelength (nm)

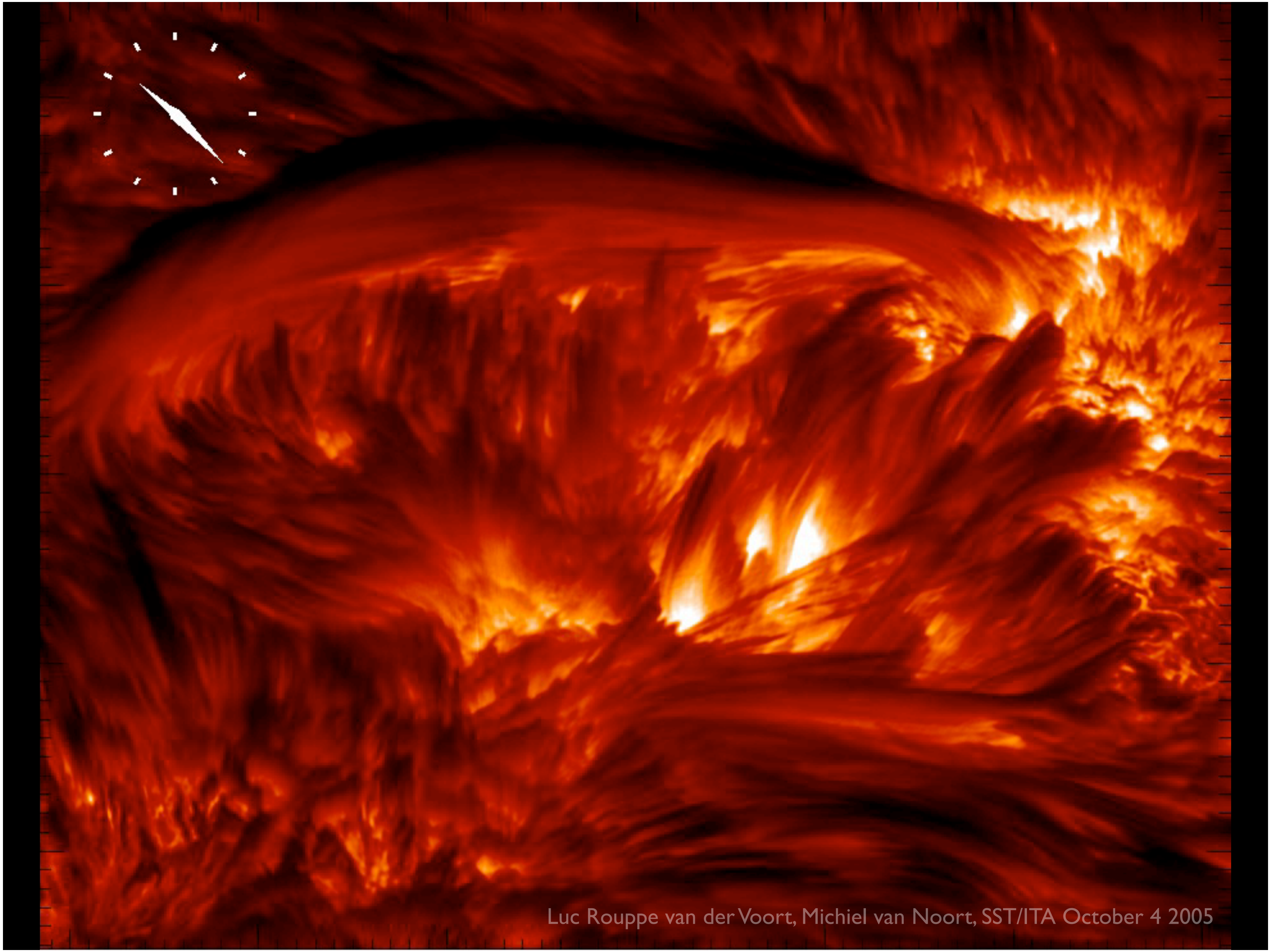


I
Q/I
Na I D1 & D2
1.6 10-5
1.4 10-5
1.2 10-5
1 10-5
8 10-6
6 10-6
4 10-6
2 10-6
0
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.005
5884
5886
5888
5890
5892
5894
5896
5898
λ (Å)

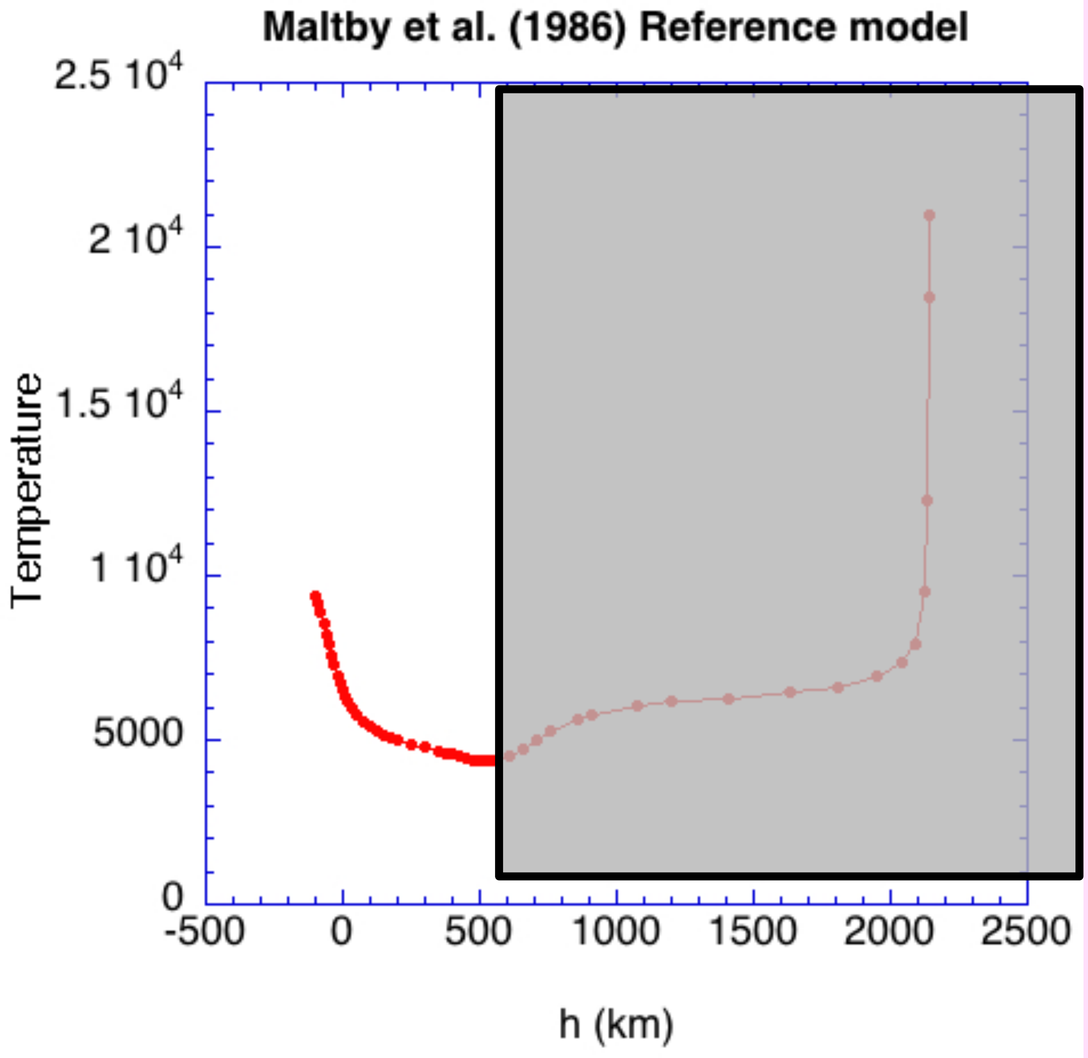
1D Atmosphere model



we limit the atmosphere model at the basis of the transition region

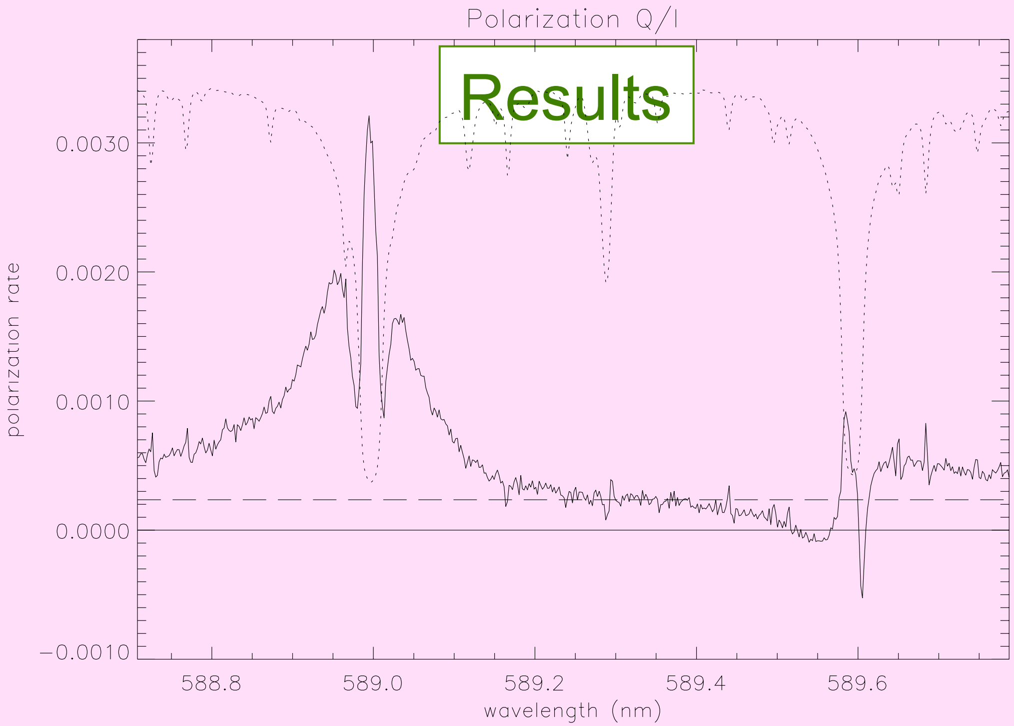

Luc Rouppe van der Voort, Michiel van Noort, SST/ITA October 4 2005

1D Atmosphere model



we limit the atmosphere model at the temperature minimum

Results



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polarization rate
588.8
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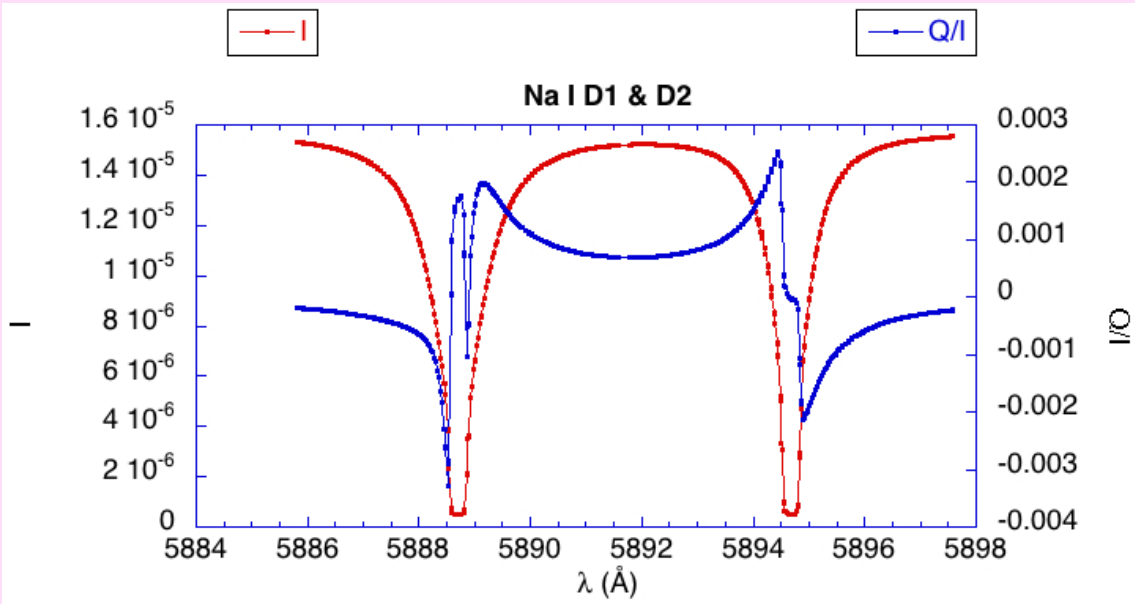
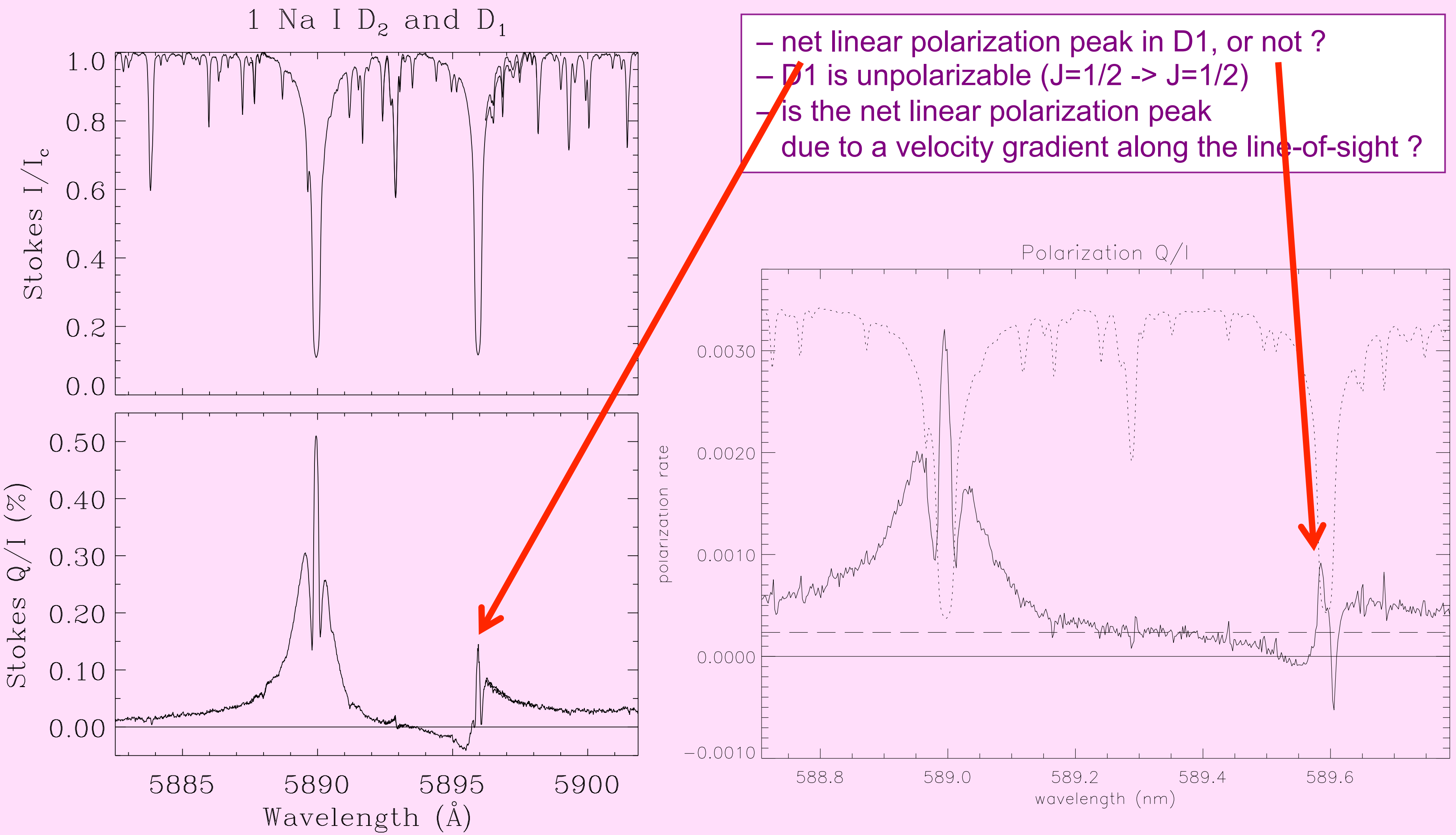
wavelength (nm)



I
Q/I
Na I D1 & D2
1.6 10-5
1.4 10-5
1.2 10-5
1 10-5
8 10-6
6 10-6
4 10-6
2 10-6
0
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
5884
5886
5888
5890
5892
5894
5896
5898
λ (Å)

Net linear polarization peak in the unpolarizable Na I D1 ?

– net linear polarization peak in D1, or not ?
– D1 is unpolarizable (J=1/2 -> J=1/2)
– is the net linear polarization peak due to a velocity gradient along the line-of-sight ?



from Stenflo & Keller, 1997, A&A, 321, 927
ZIMPOL@McMath telescope, April 1995

from Bommier & Molodij, 2002, A&A, 381, 241
THÉMIS telescope, 29 August 2000